ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტი

ფიზიკა-მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტი

მათემატიკის დეპარტამენტი

ხელნაწერის უფლებით

გიული თავდგირიძე

$\sum_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფები

მათემატიკის დოქტორის აკადემიური ხარისხის მოსაპოვებლად წარდგენილი დისერტაციის ანოტაცია

სპეციალობა-მათემატიკა

ბათუმი
2017

სადისერტაციო ნაშრომი შესრულებულია ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტის ფიზიკა-მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტის მათემატიკის დეპარტამენტში

სამეცნიერო ხელმძღვანელი:

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უცხოელი შემფასებლები:

ნეშეთ აიდინი

პროფესორი, ჩანაქკალეს 18 მარტის უნივერსიტეტი.

ალი ერდოღანი

პროფესორი, ჰაჯეტეპეს უნივერსიტეტი.

შემფასებლები:

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გულად ფარტენაძე

ფიზიკა-მათემატიკის მეცნიერებათა კანდიდატი, ასოც. პროფესორი, ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტი.

სადისერტაციო ნაშრომის დაცვა შედგება 2017 წლის 17 თებერვალს, 14^{00} საათზე, ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტის ფიზიკა-მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტის სადისერტაციო საბჭოს სხდომაზე.

მისამართი: ბათუმი, ნინოშვილის ქ.№35, უნივერსიტეტის პირველი კორპუსი, მესამე სართული, დარბაზი №55.

დისერტაციის გაცნობა შეიძლება ბათუმის შოთა რუსთაველის სახელმწიფო უნივერსიტეტის ბიბლიოთეკაში და ვებ-გვერდზე www.bsu.edu.ge

სადისერტაციო საბჭოს სწავლული მდივანი,
ასოცირებული პროფესორი დალი მახარაძე

თემის აქტუალობა

ბინარულ მიმართებათა თეორიის განვითარება დაიწყო მათემატიკური ლოგიკის ,როგორც მათემატიკის ერთ-ერთი დარგის აღმოცენებასთან ერთად და იგი ემსახურებოდა მის განვითარებას. ბინარულ მიმართებათა თეორიის საწყისი საფუძვლების განვითარებაში დიდი წვლილი მიუძღვის დე მორგანს , პირსს და ფრეგეს. შემდგომში ბინარულ მიმართებათა თეორის განვითარებაში დიდი როლი ითამაშა ფრანგმა მათემატიკოსმა რიგემ , მან ბინარულ მიმართებათა თეორია გამოიყენა დალაგებულ სიმრავლეთა შესასწავლად . ამჟამად ბინარული მიმართებები დიდ გამოყენებას პოულობს მათემატიკურ ლინგვისტიკაში, ბიოლოგიასა და მათემატიკის სხვა მრავალ დარგში, აგრეთვე ავტომატთა თეორიაში.

ბინარულ მიმართებათა თეორია მნიშვნელოვან როლს თამაშობს ალგებრის იმ დარგში რომელსაც ნაწილობრივ ასახვათა თეორიას უწოდებენ. უკანასკნელი თეორიის შესწავლამ პირველად ვაგნერი მიიყვანა ბინარულ მიმართებათა ნახევარჯგუფის ცნებამდე.

ნახევარჯგუფთა თეორიაში ცნობილია რომ ნებისმიერი ნახევარჯგუფი რომელიმე სიმრავლეზე განსაზღრული ყველა ბინარულ მიმართებათა ნახევარჯგუფის რომელიმე ქვენახევარ-ჯგუფის იზომორფულია. გამომდინარე აქედან გამართლებულია ყველა ბინარულ მიართებათა ნახევარჯგუფის რომელიმე ქვენახევარჯგუფის შესწავლა, რადგანაც ამ შემთხვევაში ფაქტიურად ხდება ბუნებაში არსებული ნახევარჯგუფების შესწავლა. ამ ნახევარჯგუფების შესწავლის ამოცანა განსაკუთ-რებით საინტერესო ხდება როცა შეისწავლება არა ცალკეული ნახევარჯგუფები, არამედ ბინარულ მიმართებათა ნახევარჯგუფის რაღაც კლასი. ჩვენ განვიხილავთ ბინარულ მიმართებათა ნახევარჯგუფების ისეთ კლასებს, რომლებიც განისაზღვრებიან გაერთიანების სრული X – ნახევარმესერთა რაღაც კლასით.

ამ მეთოდით ბინარულ მიმართებათა ნახევარჯგუფებისა და მათი ზოგიერთი მნიშვნელოვანი კლასების სისტემატური შესწავლა პირველად გამოყენებული იქნა იაშა დიასამიძის მიერ თავის სადისერტაციო ნაშრომში. ეს არის ახალი მიმართულება, რომელსაც წარმოდგენილი მონოგრაფია ეძღვნება.

ვთქვათ $\Sigma_n(X,m)$ არის m სიმძლავრის მქონე ყველა გაერთიანებათა სრული $X-$ნახევარმესერების კლასი. $D_1, D_2, ..., D_n$ ისეთი ელემენტებია $\Sigma(X,m)$ კლასიდან, რომელთაგან არცერთი ორი ერთმანეთის იზომორფული არ არის. საზოგადოდ $\Sigma_s(X,m)$ სიმბოლოთი აღინიშნება $\Sigma(X,m)$ კლასის ქვეკლასი, რომლის ყოველი ელემენტი იზომორფულია ფიქსირებული D_s $(1 \le s \le n)$ გაერთიანებათა სრული $X-$ ნახევარმესერისა.

ასევე ცნობილია, რომ თუ $\varnothing \in D$. თუ $B_X(D_1)$ და $B_X(D_2)$ ნახევარჯგუფები, რომლებიც განსაზღვრული არიან D_1 და D_2 გაერთიანებათა სრული $X-$ნახევარმესერებით, ერთმანეთის იზომორფულია, მაშინ ასევე იზომორფული იქნებიან D_1 და D_2 ნახევარმესერები, როგორც ნაწილობრივ დალაგებული სიმრავლეები თეორიულ-სიმრავლური ჩართვის მიმართ. აქედან გამომდინარეობს, რომ ნახევარჯგუფთა კლასი რომლებიც განსაზღვრული არიან $\Sigma_s(X,m)$ კლასის ნახევარმესერებით ჩაკეტილი არიან მათი იზომორფული სახეების მიმართ. მათი აბსტრაქტული თვისებები (ე.ი. ნახევარჯგუფთა ისეთი თვისებები, რომლებიც შეინახებიან მათი იზომორფიზმების დროს) ძირითადად განისაზღვრებიან D ნახევარმესერის თვისებებით. აქედან გამომდინარე ნახევარჯგუფებისა და ნახევარჯგუფთა კლასების შესწავლის აღნიშნული მეთოდი საკმაოდ პერსპექტიული და ეფექტურია.

შრომის მიზანი

სადისერტაციო ნაშრომის ძირითადი მიზანია იმ ბინარულ მიმართებათა სრული ნახევარჯგუფების აბსტრაქტული თვისებების შესწავლა, რომლებიც განსაზღვრული არიან $\Sigma_3(X,8)$ კლასის გაერთიანებათა სრული $X-$ ნახევარმესერებით.

კვლევის საგანია როგორც $\Sigma_3(X,8)$ კლასის გაერთიანებათა სრული $X-$ ნახევარმესერები, ისე ამ ნახევარმესერებით განსაზღვრული ყველა ბინარულ მიმართებათა სრული ნახევარჯგუფები, რადგანაც მოცემული ნახევარმესერები, როგორც

გამოკვლევები გვიჩვენებენ, ატარებენ მნიშვნელოვან ინფორმაციას იმ ბინარულ მიმართებათა სრული ნახევარჯგუფების შესახებ, რომლებსაც ისინი განსაზღვრავენ.

შრომის მეცნიერული სიახლე

ნაშრომში პირველად განიხილება ბინარულ მიმართებათა იმ სრული $B_X(D)$ ნახევარჯგუფის კლასები, რომლებიც განსაზღვრულია $\Sigma_3(X,8)$ კლასის ნახევარმესერებით. ნახევარჯგუფთა კლასისა და ამ კლასის თითოეული ელემენტის შესწავლა ხდება გაერთიანების სრულ X-ნახევარმესერის თვისებებზე დაყრდნობით. ნაშრომში მიღებული შედეგები საშუალებას გვაძლევს ამ ნახევარჯგუფის განმსაზღვრელი ნახევარმესერს დიაგრამაზე დაყრდნობით ვილაპარაკოთ მოცემული ნახევარჯგუფის დამახასიათებელ მრავალ თვისებაზე, კერძოდ გააჩნიათ თუ არა მარჯვენა ერთეულები, როგორი სახე ექნება მათ იდემპოტენტურ ელემენტებს, მაქსიმალურ ქვეჯგუფებს, რეგულარულ ელემენტებს და კიდევ სხვა მრავალ ფაქტზე.

ნაშრომში აღწერილია $\Sigma_3(X,8)$ კლასის ყველა ქვენახევარმესერები და გამოყოფილია მათგან $XI-$ქვენახევარმესერები. როცა X სასრული სიმრავლეა, გამოყვანილია $\Sigma_3(X,8)$ კლასში ნახევარმესერების რაოდენობის დათვლის ფორმულები.

აღწერილია $\Sigma_3(X,8)$ კლასის ყოველი D ნახევარმესერით განსაზღვრული $B_X(D)$ ნახევარჯგუფის იდემპოტენტური ელემენტები და რეგულარული ელემენტები როცა X სასრული სიმრავლეა, გამოყვანილია რაოდენობის დათვლის ფორმულები. აგრეთვე აღწერილია $\Sigma_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული $B_X(D)$ ნახევარჯგუფის მაქსიმალური ქვეჯგუფები.

გამოკვლევის ძირითადი მეთოდები

როგორც ცნობილია ნახევარჯგუფის იდემპოტენტური ელემენტები, ცალმხრივი ერთეულები, რეგულარული ელემენტები, მაქსიმალური ქვეჯგუფები მნიშვნელოვან როლს ასრულებენ ნახევარჯგუფთა აბსტრაქტული თვისებების შესწავლაში. გამომდინარე აქედან, ბინარულ მიმართებათა სრული

ნახევარჯგუფების კლასების შესწავლისას დიდი ყურადღება ექცევა ნახევარჯგუფში ზემოთ ჩამოთვლილი საკითხების შესწავლას.

სადისერტაციო ნაშრომში ნახევარჯგუფთა თვისებების შესწავლა ხდება ნახევარმესერთა თვისებების საშუალებით. ბინარულ მიმართებათა სრული ნახევარჯგუფების აბსტრაქტული თვისებების მნიშვნელოვანი ნაწილი ძირითადად განისაზღვრება იმ გაერთიანებათა სრული $X-$ნახევარმესერებით, რომლებიც განსაზღვრავენ მოცემული კლასის ელემენტებს. ნაშრომში გამოიყენება მესერთა თეორიის, ნახევარჯგუფთა თეორიის, სიმრავლეთა თეორიის და კომბინატორიკის ზოგადი მეთოდი.

ნაშრომის პრაქტიკული და თეორიული ღირებულება

სადისერტაციო ნაშრომი ატარებს ძირითადად თეორიულ ხასიათს. ნაშრომში მიღებული შედეგები შეიძლება შემდგომ გამოყენებული იქნას ნახევარჯგუფთა და ნახევარმესერთა გამოკვლევებში.

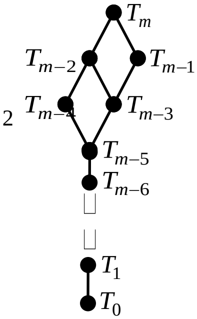
დისერტაციის სტრუქტურა და მოცულობა

სადისერტაციო ნაშრომი შედგება შესავლის 8 პარაგრაფისა და 3 დანართისაგან. ნაშრომის საერთო მოცულობა არის კომპიუტერით დაკაბადონებული 251 გვერდი. (მათგან 181 გვერდი ნაშრომის ძირითადი ნაწილია, დანარჩენი კი დამატებები).

ნაშრომის საერთო დახასიათება

1. ზოგიერთი სახის ნახევარმესერებით განსაზღვრული ბინარულ მიმართებათა ნახევარჯგუფების რეგულარული და იდემპოტენტური ელემენტები.მოცემულ პარაგრაფში აღწერილია რამოდენიმე სახის ნახევარმესერებით განსაზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფების რეგულარული და იდემპოტენტური ელემენტები. გამოყვანილია მათი რაოდენობის დასათვლელი ფორმულები, რომლებიც შემდგომში გვჭირდება $\sum_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული ნახევარჯგუფების შესასწავლად.

ლემა1.1. ვთქვათ $Q=\{T_0,T_1,...,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ არის გაერთიანებათა X ნახევარმესერი(იხ. ნახ. 1), მაშინ Q ყოველთვის იქნება გაერთიანებათა XI – ნახევარმესერი.



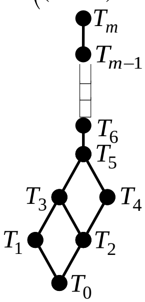
ნახ1.1

თეორემა 1.1. ვთქვათ $Q=\{T_0,T_1,...,T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ არის გაერთიანებათა X ნახევარმესერი (იხ.ნახ.1), თუ Q გაერთიანებათა XI – ნახევარმესერი და $D'=\{\bar{T}_0,\bar{T}_1,...,\bar{T}_{m-4},\bar{T}_{m-3},\bar{T}_{m-2},\bar{T}_{m-1},\bar{T}_m\}$ არიან α – იზომორფული და $|\Omega(Q)|=m_0$, მაშინ სამართლიანი იქნება შემდეგი ტოლობა :

$$|R(D')|=m_0\cdot\left(2^{|\bar{T}_1\setminus\bar{T}_0|}-1\right)\cdot\left(3^{|\bar{T}_2\setminus\bar{T}_1|}-2^{|\bar{T}_2\setminus\bar{T}_1|}\right)\cdots\left((m-4)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-5}|}-(m-5)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-5}|}\right)\cdot$$
$$\cdot(m-4)^{|((\bar{T}_{m-1}\cap\bar{T}_{m-4})\setminus\bar{T}_{m-5})|}\cdot\left((m-4)^{|\bar{T}_{m-5}\setminus\bar{T}_{m-6}|}-(m-5)^{|\bar{T}_{m-5}\setminus\bar{T}_{m-6}|}\right)\cdot$$
$$\cdot\left((m-3)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-1}|}-(m-4)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-1}|}\right)\cdot\left((m-3)^{|\bar{T}_{m-3}\setminus\bar{T}_{m-4}|}-(m-4)^{|\bar{T}_{m-3}\setminus\bar{T}_{m-4}|}\right)\cdot$$
$$\cdot\left((m-2)^{|\bar{T}_{m-1}\setminus\bar{T}_{m-2}|}-(m-3)^{|\bar{T}_{m-1}\setminus\bar{T}_{m-2}|}\right)\cdot(m+1)^{|X\setminus\bar{T}_m|}.$$

შედეგი 1.1. ვთქვათ $Q=\{T_0,T_1,...,T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ არის გაერთიანებათა X ნახევარმესერი. თუ $E_X^{(r)}(Q)$ არის $B_X(Q)$ ნახევარჯგუფის ყველა მარჯვენა ერთეულების სიმრავლე მაშინ ის გამოითვლება შემდეგი ფორმულით

$$\left|E_X^{(r)}(Q)\right|=\left(2^{|T_1\setminus T_0|}-1\right)\cdot\left(3^{|T_2\setminus T_1|}-2^{|T_2\setminus T_1|}\right)\cdots\left((m-4)^{|T_{m-4}\setminus T_{m-5}|}-(m-5)^{|T_{m-4}\setminus T_{m-5}|}\right)\cdot$$
$$\cdot(m-4)^{|((T_{m-1}\cap T_{m-4})\setminus T_{m-5})|}\cdot\left((m-4)^{|T_{m-5}\setminus T_{m-6}|}-(m-5)^{|T_{m-5}\setminus T_{m-6}|}\right)\cdot$$
$$\cdot\left((m-3)^{|T_{m-4}\setminus T_{m-1}|}-(m-4)^{|T_{m-4}\setminus T_{m-1}|}\right)\cdot\left((m-3)^{|T_{m-3}\setminus T_{m-4}|}-(m-4)^{|T_{m-3}\setminus T_{m-4}|}\right)\cdot$$
$$\cdot\left((m-2)^{|T_{m-1}\setminus T_{m-2}|}-(m-3)^{|T_{m-1}\setminus T_{m-2}|}\right)\cdot(m+1)^{|X\setminus T_m|}.$$



ნახ1.2

ლემა1.2. ვთქვათ $Q=\{T_0,T_1,...,T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ არის გაერთიანებათა X ნახევარმესერი (იხ. ნახ. 2), მაშინ Q ყოველთვის იქნება გაერთიანებათა XI – ნახევარმესერი.

თეორემა 1. 2. ვთქვათ $Q=\{T_0,T_1,T_2,T_3,...,T_m\}$ $(m\geq 6)$ არის გაერთიანებათა X ნახევარმესერი (იხ. ნახ. 2), თუ Q გაერთიანებათა XI – ნახევარმესერი და $D'=\{\bar{T}_0,\bar{T}_1,\bar{T}_2,...,\bar{T}_m\}$ არიან α – იზომორფული და $|\Omega(Q)|=m_0$, მაშინ

სამართლიანია შემდეგი ტოლობა:

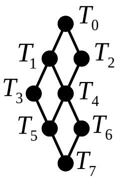
$$\left|R\left(D'\right)\right| = m_0 \cdot \left(3^{\left|\bar{T}_4 \setminus \bar{T}_3\right|} - 2^{\left|\bar{T}_4 \setminus \bar{T}_3\right|}\right) \cdot \left(2^{\left|\bar{T}_1 \setminus \bar{T}_4\right|} - 1\right) \cdot \left(2^{\left|\bar{T}_2 \setminus \bar{T}_1\right|} - 1\right) \cdot \left(7^{\left|\bar{T}_6 \setminus \bar{T}_5\right|} - 6^{\left|\bar{T}_6 \setminus \bar{T}_5\right|}\right) \cdot$$
$$\cdot \left(8^{\left|\bar{T}_7 \setminus \bar{T}_6\right|} - 7^{\left|\bar{T}_7 \setminus \bar{T}_6\right|}\right) \cdots \left(m^{\left|\bar{T}_{m-1} \setminus \bar{T}_{m-2}\right|} - (m-1)^{\left|\bar{T}_{m-1} \setminus \bar{T}_{m-2}\right|}\right) \cdot (m+1)^{\left|X \setminus \bar{T}_m\right|}.$$

შედეგი 1.2. ვთქვათ $Q=\{T_0,T_1,T_2,T_3,...,T_m\}$ $(m \geq 6)$ გაერთიანებათა ნახევარმესერია, თუ $E_X^{(r)}(Q)$ არის $B_X(Q)$,ნახევარჯგუფის ყველა მარჯვენა ერთეულების სიმრავლე, მაშინ სამართლიანია შედეგი ფორმულა

$$\left|E_X^{(r)}(Q)\right| = \left(3^{\left|T_1 \setminus T_3\right|} - 2^{\left|T_4 \setminus T_3\right|}\right) \cdot \left(2^{\left|T_1 \setminus T_4\right|} - 1\right) \cdot \left(2^{\left|T_2 \setminus T_1\right|} - 1\right) \cdot \left(7^{\left|T_6 \setminus T_5\right|} - 6^{\left|T_6 \setminus T_5\right|}\right) \cdot$$
$$\cdot \left(8^{\left|T_7 \setminus T_6\right|} - 7^{\left|T_7 \setminus T_6\right|}\right) \cdots \left(m^{\left|T_{m-1} \setminus T_{m-2}\right|} - (m-1)^{\left|T_{m-1} \setminus T_{m-2}\right|}\right) \cdot (m+1)^{\left|X \setminus T_m\right|}.$$

ლემა 1.3. ვთქვათ $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\} \in \sum_3(X,8)$ არის გაერ-თიანებათა X ნახევარმესერი (იხ. ნახ. 3), მაშინ Q ყოველთვის იქნება გაერთიანებათა XI – ნახევარმესერი.

თეორემა 1.3. ვთქვათ $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\} \in \sum_3(X,8)$ და $\left|\Sigma_3(X,8)\right| = m_0$ თუ X სასრულო სიმრავლეა და Q გაერთიანებათა XI ნახევარ-მესერი და $Q'=\{\bar{T}_7,\bar{T}_6,\bar{T}_5,\bar{T}_4,\bar{T}_3,\bar{T}_2,\bar{T}_1\}$ არიან α -იზომორფული მაშინ სამართლიანია იქნება შემდეგი ტოლობა:



ნახ1.3

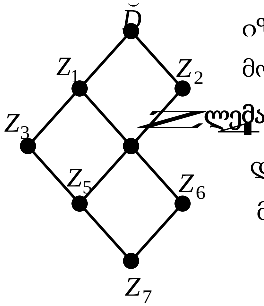
$$\left|R\left(Q'\right)\right| = m_0 \cdot \left(2^{\left|\bar{T}_6 \setminus \bar{T}_3\right|} - 1\right) \cdot 2^{\left|\left(\bar{T}_3 \cap \bar{T}_2\right) \setminus \bar{T}_4\right|} \cdot \left(2^{\left|\bar{T}_5 \setminus \bar{T}_6\right|} - 1\right) \cdot$$
$$\cdot \left(3^{\left|\bar{T}_3 \setminus \bar{T}_2\right|} - 2^{\left|\bar{T}_3 \setminus \bar{T}_2\right|}\right) \cdot \left(5^{\left|\bar{T}_2 \setminus \bar{T}_1\right|} - 4^{\left|\bar{T}_2 \setminus \bar{T}_1\right|}\right) \cdot 8^{\left|X \setminus \bar{T}_0\right|}$$

შედეგი 1.3. ვთქვათ $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\} \in \sum_3(X,8)$, არის გაერთიანებათა X ნახევარმესერი, თუ $E_X^{(r)}(Q)$ არის $B_X(Q)$ ნახევარჯგუფის ყველა მარჯვენა ერთეულების სიმრავლე მაშინ ის გამოითვლება შემდეგი ფორმულით

$$\left|E_X^{(r)}(Q)\right| = \left(2^{\left|T_6 \setminus T_3\right|} - 1\right) \cdot 2^{\left|\left(T_3 \cap T_2\right) \setminus T_4\right|} \cdot \left(2^{\left|T_5 \setminus T_6\right|} - 1\right) \cdot$$
$$\cdot \left(3^{\left|T_3 \setminus T_2\right|} - 2^{\left|T_3 \setminus T_2\right|}\right) \cdot \left(5^{\left|T_2 \setminus T_1\right|} - 4^{\left|T_2 \setminus T_1\right|}\right) \cdot 8^{\left|X \setminus T_0\right|}.$$

2. $\Sigma_3(X,8)$ კლასის ნახევარმესერების ქვენახევარმესერები. მოცემულ პარაგრაფში აღწერილია $\Sigma_3(X,8)$ კლასის ნახევარ-მესერები და მათგან გამოყოფილია ისინი რომლებიც არიან $XI-$ ნახევარმესერები. $\Sigma_3(X,8)$ სიმბოლოთი ავღნიშნოთ გაერ-თიანების $X-$ ნახევარმესერთა კლასი, რომლის ყოველი ელემენტი რომელიღაც $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$ გაერთიანების X ნახევარმესერის იზომორფულია. აღნიშნული ნახევარმესერი მოცემულია ნახაზი 2.1.-ზე.

ლემა 2.1. ვთქვათ $D\in\Sigma_3(X,8)$, $|\Sigma_3(X,8)|=s$ და $|X|\ge\delta\ge 4$.თუ X სასრულო სიმრავლეა მაშინ $s=5^n-4\cdot 6^n+6\cdot 7^n-4\cdot 8^n+9^n$



ნახაზი.2.1

მაგალითი 2.1. ვთქვათ $n=4,5,6,7,8$ მაშინ:

$s=48,1680,35520,294000,4198824$

$|B_X(D)|=4096,\ 32768,\ 262144,\ 2097152,\ 16777216.$

მოცემული რიცხვები გვიჩვენებს, რომ მაგალითად, თუ $|X|=8$, მაშინ მოცემული კლასის ყველა ნახევარჯგუფთა ელემენტების რიცხვი 4198824 ის ტოლია, ხოლო ელემენტების რიცხვი თითოეულ ნახევარჯგუფში, რომელიც მოცემულ კლასს მიეკუთვნება 16777216-ის ტოლია.

ახლა აღვწეროთ $\Sigma_3(X,8)$ კლასის ყველა XI -ქვენახევარმესერი.

ლემა 2.2. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. მაშინ შემდეგი სიმრავლეებით ამოიწურებიან D ნახევარმესერის ყველა XI ქვენახევარმესერი:

1) $\{Z_7\},\{Z_6\},\{Z_5\},\{Z_4\},\{Z_3\},\{Z_2\},\{Z_1\},\{\breve{D}\}$; (იხ.დიაგრამა 1 ნახაზი 2.2-ზე);

2) $\{Z_7,Z_6\},\{Z_7,Z_5\},\{Z_7,Z_4\},\{Z_7,Z_3\},\{Z_7,Z_2\},\{Z_7,Z_1\},\{Z_7,\breve{D}\},\{Z_6,Z_4\},$
$\{Z_6,Z_2\},\{Z_6,Z_1\},\{Z_6,\breve{D}\},\{Z_5,Z_4\},\{Z_5,Z_3\},\{Z_5,Z_2\},\{Z_5,Z_1\},\{Z_5,\breve{D}\},$
$\{Z_4,Z_2\},\{Z_4,Z_1\},\{Z_4,\breve{D}\},\{Z_3,Z_1\},\{Z_3,\breve{D}\},\{Z_2,\breve{D}\},\{Z_1,\breve{D}\};$

(იხ.დიაგრამა 2 ნახაზი 2.2-ზე);

3) $\{Z_7,Z_6,Z_4\},\{Z_7,Z_6,Z_2\},\{Z_7,Z_6,Z_1\},\{Z_7,Z_6,\breve{D}\},\{Z_7,Z_5,Z_4\},\{Z_7,Z_5,Z_3\},$
$\{Z_7,Z_5,Z_2\},\{Z_7,Z_5,Z_1\},\{Z_7,Z_5,\breve{D}\},\{Z_7,Z_4,Z_2\},\{Z_7,Z_4,Z_1\},\{Z_7,Z_4,D\},$
$\{Z_7,Z_3,Z_1\},\{Z_7,Z_3,\breve{D}\},\{Z_7,Z_2,\breve{D}\},\{Z_7,Z_1,\breve{D}\},\{Z_6,Z_4,Z_2\},\{Z_6,Z_4,Z_1\},$
$\{Z_6,Z_4,\breve{D}\},\{Z_6,Z_2,\breve{D}\},\{Z_6,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2\},\{Z_5,Z_4,Z_1\},\{Z_5,Z_4,\breve{D}\},$
$\{Z_5,Z_3,Z_1\},\{Z_5,Z_3,\breve{D}\},\{Z_5,Z_2,\breve{D}\},\{Z_5,Z_1,\breve{D}\},\{Z_4,Z_2,\breve{D}\},\{Z_4,Z_1,\breve{D}\},$
$\{Z_3,Z_1,\breve{D}\};$

(იხ.დიაგრამა 3 ნახაზი 2.2-ზე);

4) $\{Z_7,Z_6,Z_4,Z_2\},\{Z_7,Z_6,Z_4,Z_1\},\{Z_7,Z_6,Z_4,\breve{D}\},\{Z_7,Z_6,Z_2,D\},\{Z_7,Z_6,Z_1,\breve{D}\},$
$\{Z_7,Z_5,Z_4,Z_2\},\{Z_7,Z_5,Z_4,Z_1\},\{Z_7,Z_5,Z_4,\breve{D}\}\ \{Z_7,Z_5,Z_3,Z_1\},\{Z_7,Z_5,Z_3,\breve{D}\},$
$\{Z_7,Z_5,Z_2,\breve{D}\},\{Z_7,Z_5,Z_1,\breve{D}\},\{Z_7,Z_4,Z_2,D\},\{Z_7,Z_4,Z_1,\breve{D}\},\{Z_7,Z_3,Z_1,\breve{D}\},$
$\{Z_6,Z_4,Z_2,\breve{D}\},\{Z_6,Z_4,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2,\breve{D}\},\{Z_5,Z_4,Z_1,\breve{D}\},\{Z_5,Z_3,Z_1,\breve{D}\};$

(იხ.დიაგრამა 4 ნახაზი2.2-ზე);

5) $\{Z_7,Z_6,Z_4,Z_2,\breve{D}\},\{Z_7,Z_6,Z_4,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,\breve{D}\},\{Z_7,Z_5,Z_4,Z_1,\breve{D}\},\{Z_7,Z_5,Z_3,Z_1,\breve{D}\},$

(იხ.დიაგრამა 5 ნახაზი 2.2-ზე);

6) $\{Z_7,Z_6,Z_5,Z_4\},\{Z_7,Z_6,Z_3,Z_1\},\{Z_7,Z_4,Z_3,Z_1\},\{Z_7,Z_3,Z_2,\breve{D}\},\{Z_7,Z_2,Z_1,\breve{D}\},$
$\{Z_6,Z_2,Z_1,\breve{D}\},\{Z_5,Z_4,Z_3,Z_1\},\{Z_5,Z_3,Z_2,\breve{D}\},\{Z_5,Z_2,Z_1,\breve{D}\},\{Z_4,Z_2,Z_1,\breve{D}\};$

(იხ.დიაგრამა 6 ნახაზი 2.2-ზე);

7) $\{Z_7,Z_6,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_3,Z_1,\},\{Z_7,Z_5,Z_3,Z_2,\breve{D}\},\{Z_7,Z_5,Z_2,Z_1,\breve{D}\},$
$\{Z_7,Z_4,Z_2,Z_1,\breve{D}\},\{Z_6,Z_4,Z_2,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2,Z_1,\breve{D}\}$

(იხ.დიაგრამა 7 ნახაზი 2.2-ზე);

8) $\{Z_7,Z_6,Z_4,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,Z_1,\breve{D}\};$ (იხ.დიაგრამა 8 ნახაზი3.4-ზე)

9) $\{Z_7,Z_5,Z_4,Z_3,Z_1,\breve{D}\};$ (იხ.დიაგრამა 9 ნახაზი 2.2-ზე);

10) $\{Z_7,Z_6,Z_5,Z_4,Z_2\},\{Z_7,Z_6,Z_5,Z_4,Z_1\},\{Z_7,Z_6,Z_5,Z_4,\breve{D}\},\{Z_7,Z_4,Z_3,Z_1,\breve{D}\},$

$\{Z_7,Z_6,Z_3,Z_1,\breve{D}\},\{Z_5,Z_4,Z_3,Z_1,\breve{D}\}$ (იხ.დიაგრამა 10 ნახაზი 2.2-ზე);

11) $\{Z_7,Z_6,Z_5,Z_4,Z_2,\breve{D}\},\{Z_7,Z_6,Z_5,Z_4,Z_1,\breve{D}\};$ (იხ.დიაგრამა 11 ნახაზი 2.2-ზე);

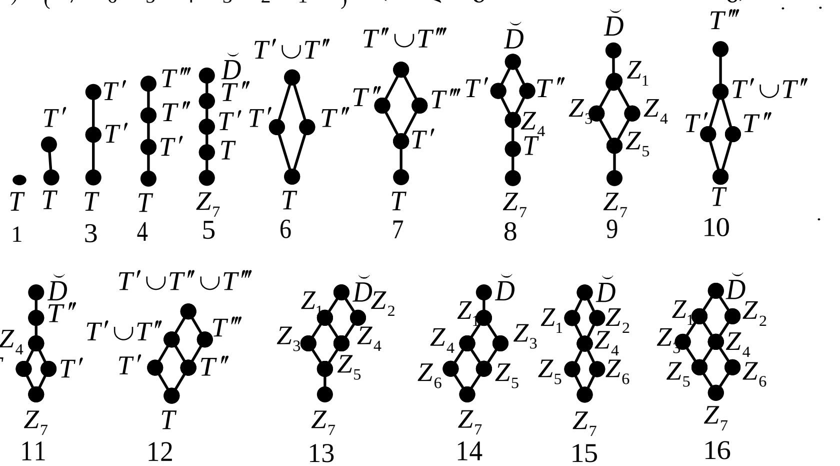
12) $\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1\},\{Z_7,Z_6,Z_3,Z_2,Z_1,\breve{D}\},\{Z_7,Z_4,Z_3,Z_2,Z_1,\breve{D}\},$
$\{Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$; (იხ.დიაგრამა 12 ნახაზი 2.2-ზე);

13) $\{Z_7,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$; (იხ.დიაგრამა 13 ნახაზი 2.2-ზე);

14) $\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1,\breve{D}\}$; (იხ.დიაგრამა 14 ნახაზი 2.2-ზე);

15) $\{Z_7,Z_6,Z_5,Z_4,Z_2,Z_1,\breve{D}\}$; (იხ.დიაგრამა 15 ნახაზი2.2-ზე);

16) $\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$; (იხ.დიაგრამა 16 ნახაზი2.2-ზე);



ნახ.2.2

3. $\Sigma_3(X,8)$ **კლასის ნახევარმესერებით განსაზღვრული** $B_X(D)$ **ნახევარჯგუფის იდეპოტენტური ელემენტები, როცა** $Z_7 \neq \varnothing$. მოცემულ პარაგრაფში განხილულია $\Sigma_3(X,8)$ კლასით განსაზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფები და აღწერილია მოცემული ნახევარჯგუფის იდემპოტენტური ელემენტები. განხილულია ის შემთხვევა როცა X სასრულო სიმრავლეა და $Z_7 \neq \varnothing$. გამოყვანილია ფორმულები, რომელთა საშუალებითაც დათვლილია მოცემული ნახევარჯგუფის იდემპოტენტური ელემენტების რაოდენობა.

ვთქვათ $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$.

Q_i $\left(i=1,2,3,\ldots,16\right)$ სიმბოლოებით აღვნიშნოთ შემდეგი სიმრავლეები:

1) $Q_1=\left\{T\right\}$, სადაც $T\in D$ (იხ. დიაგრამა 1 ნახაზი 2.2-ზე)

2) $Q_2=\left\{T,T'\right\}$, სადაც $T,T'\in D$, $T\subset T'$; (იხ. დიაგრამა 2 ნახაზი 2.2-ზე)

3) $Q_3=\left\{T,T',T''\right\}$, სადაც $T,T',T''\in D$, $T\subset T'\subset T''$; (იხ. დიაგრამა 3 ნახაზი 2.2-ზე)

4) $Q_4=\left\{T,T',T'',T'''\right\}$, სადაც $T,T',T'',T'''\in D$, $T\subset T'\subset T''\subset T'''$,(იხ. დიაგრამა 4 ნახ 2.2-ზე)

5) $Q_5=\left\{Z_7,T,T',T'',\breve{D}\right\}$, სადაც $Z_7,T,T',T'',\breve{D}\in D,\ Z_7\subset T\subset T'\subset T''\subset\breve{D}$ (იხ. დიაგრამა 5 ნახაზი 2.2-ზე)

6) $Q_6=\left\{T,T',T'',T'\cup T''\right\}$, სადაც $T,T',T''\in D$, $T\subset T'$,$T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$ (იხ. დიაგრამა 6 ნახაზი 2.2-ზე)

7) $Q_7=\left\{T,T',T'',T''',T''\cup T'''\right\}$, სადაც $T\subset T'\subset T''$, $T\subset T'\subset T'''$, $T''\setminus T'''\neq\varnothing$, $T'''\setminus T''\neq\varnothing$ (იხ. დიაგრამა 7 ნახაზი 2.2-ზე)

8) $Q_8=\left\{Z_7,T,Z_4,Z_2,Z_1,\breve{D}\right\}$, სადაც $T\in\left\{Z_6,Z_5\right\}$ (იხ. დიაგრამა 8 ნახაზი 2.2-ზე)

9) $Q_9=\left\{Z_7,Z_5,Z_4,Z_3,Z_1,\breve{D}\right\}$, (იხ. დიაგრამა 9 ნახაზი 2.2-ზე)

10) $Q_{10}=\left\{T,T',T'',T'\cup T'',T'''\right\}$, სადაც $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T'\cup T''\subset T'''$ (იხ. დიაგრამა 10 ნახაზი 2.2-ზე)

11) $Q_{11}=\left\{Z_7,Z_6,Z_5,Z_4,T,\breve{D}\right\}$, სადაც $T\in\left\{Z_2,Z_1\right\}$, (იხ. დიაგრამა 11 ნახაზი 2.2-ზე)

12) $Q_{12}=\left\{T,T',T'',T'\cup T'',T''',T'\cup T''\cup T'''\right\}$, სადაც $T\subset T'$,$T\subset T''$,$T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset T'''$, $\left(T'\cup T''\right)\setminus T'''\neq\varnothing$, $T'''\setminus\left(T'\cup T''\right)\neq\varnothing$, (იხ. დიაგრამა 12 ნახაზი 3.4-ზე)

13) $Q_{13}=\left\{Z_7,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}$, (იხ. დიაგრამა 13 ნახაზი 2.2-ზე)

14) $Q_{14}=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_1,\breve{D}\right\}$, (იხ. დიაგრამა 14 ნახაზი 2.2-ზე)

15) $Q_{15}=\left\{Z_7,Z_6,Z_5,Z_4,Z_2,Z_1,\breve{D}\right\}$ (იხ. დიაგრამა 15 ნახაზი 2.2-ზე)

16) $Q_{16}=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}$ (იხ. დიაგრამა 16 ნახაზი 2.2-ზე).

თეორემა 3.1. ვთქვათ $D\in\Sigma_3\left(X,8\right)$, $Z_7\neq\varnothing$ და $\alpha\in B_X\left(D\right)$. $B_X\left(D\right)$ ნახევარჯგუფის α ბინარული მიმართება არის მოცემული ნახევარჯგუფის იდემპოტენტური ელემენტი მაშინ და მხოლოდ მაშინ, როცა α ბინარული მიმართება აკმაყოფილებს შემდეგი პირობებიდან ერთ-ერთს:

1) $\alpha=X\times T$, სადაც $T\in D$;

2) $\alpha=\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)$, სადაც $T,T'\in D$, $T\subset T'$, $Y_T^{\alpha},Y_{T'}^{\alpha}\notin\left\{\varnothing\right\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_T^{\alpha}\supseteq T$, $Y_{T'}^{\alpha}\cap T'\neq\varnothing$;

3) $\alpha=\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)\cup\left(Y_{T''}^{\alpha}\times T''\right)$, სადაც $T,T',T''\in D$, $T\subset T'\subset T''$, $Y_T^{\alpha},Y_{T'}^{\alpha},Y_{T''}^{\alpha}\notin\left\{\varnothing\right\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_T^{\alpha}\supseteq T$, $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\supseteq T'$, $Y_{T'}^{\alpha}\cap T'\neq\varnothing$, $Y_{T''}^{\alpha}\cap T''\neq\varnothing$;

4) $\alpha=\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)\cup\left(Y_{T''}^{\alpha}\times T''\right)\cup\left(Y_{T'''}^{\alpha}\times T'''\right)$, სადაც $T,T',T'',T'''\in D$, $T\subset T'\subset T''\subset T'''$, $Y_T^{\alpha},Y_{T'}^{\alpha},Y_{T''}^{\alpha},Y_{T'''}^{\alpha}\notin\left\{\varnothing\right\}$, და აკმაყოფილებს შემდეგ პირობებს: $Y_T^{\alpha}\supseteq T$, $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\supseteq T'$, $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\cup Y_{T''}^{\alpha}\supseteq T''$, $Y_{T'}^{\alpha}\cap T'\neq\varnothing$, $Y_{T''}^{\alpha}\cap T''\neq\varnothing$, $Y_{T'''}^{\alpha}\cap T'''\neq\varnothing$;

5) $\alpha=\left(Y_7^{\alpha}\times Z_7\right)\cup\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)\cup\left(Y_{T''}^{\alpha}\times T''\right)\cup\left(Y_0^{\alpha}\times\breve{D}\right)$, სადაც $Z_7\subset T\subset T'\subset T''\subset\breve{D}$ $Y_7^{\alpha},Y_T^{\alpha},Y_{T'}^{\alpha},Y_{T''}^{\alpha},Y_0^{\alpha}\notin\left\{\varnothing\right\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_7^{\alpha}\supseteq Z_7$, $Y_7^{\alpha}\cup Y_T^{\alpha}\supseteq T$, $Y_7^{\alpha}\cup Y_T^{\alpha}\cup Y_{T'}^{\alpha}\supseteq T'$, $Y_7^{\alpha}\cup Y_T^{\alpha}\cup Y_{T'}^{\alpha}\cup Y_{T''}^{\alpha}\supseteq T''$, $Y_T^{\alpha}\cap T\neq\varnothing$, $Y_{T'}^{\alpha}\cap T'\neq\varnothing$, $Y_{T''}^{\alpha}\cap T''\neq\varnothing$, $Y_0^{\alpha}\cap\breve{D}\neq\varnothing$;

6) $\alpha=\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)\cup\left(Y_{T''}^{\alpha}\times T''\right)\cup\left(Y_{T'\cup T''}^{\alpha}\times\left(T'\cup T''\right)\right)$, სადაც $T,T',T''\in D$, $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $Y_T^{\alpha},Y_{T'}^{\alpha},Y_{T''}^{\alpha}\notin\left\{\varnothing\right\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\supseteq T'$, $Y_T^{\alpha}\cup Y_{T''}^{\alpha}\supseteq T''$, $Y_{T'}^{\alpha}\cap T'\neq\varnothing$, $Y_{T''}^{\alpha}\cap T''\neq\varnothing$;

7) $\alpha=\left(Y_T^{\alpha}\times T\right)\cup\left(Y_{T'}^{\alpha}\times T'\right)\cup\left(Y_{T''}^{\alpha}\times T''\right)\cup\left(Y_{T'''}^{\alpha}\times T'''\right)\cup\left(Y_{T''\cup T'''}^{\alpha}\times\left(T''\cup T'''\right)\right)$, სადაც $T\subset T'\subset T''$, $T\subset T'\subset T'''$, $T''\setminus T'''\neq\varnothing$, $T'''\setminus T''\neq\varnothing$ $Y_T^{\alpha},Y_{T'}^{\alpha},Y_{T''}^{\alpha},Y_{T'''}^{\alpha}\notin\left\{\varnothing\right\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^{\alpha}\supseteq T$ $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\supseteq T'$, $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\cup Y_{T''}^{\alpha}\supseteq T''$, $Y_T^{\alpha}\cup Y_{T'}^{\alpha}\cup Y_{T'''}^{\alpha}\supseteq T'''$ $Y_{T'}^{\alpha}\cap T'\neq\varnothing$, $Y_{T''}^{\alpha}\cap T''\neq\varnothing$, $Y_{T'''}^{\alpha}\cap T'''\neq\varnothing$.

8) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$,

სადაც $T\in\{Z_6,Z_5\}$, $Y_7^\alpha,Y_T^\alpha,Y_4^\alpha,Y_2^\alpha,Y_1^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_T^\alpha\supseteq T$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_T^\alpha\cap T\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$ $Y_2^\alpha\cap Z_2\neq\varnothing$, $Y_1^\alpha\cap Z_1\neq\varnothing$

9) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$,

სადაც, $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

10) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)$,

სადაც $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T'\cup T''\subset T'''$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_{T'}^\alpha\cap T'\neq\varnothing$ $Y_{T''}^\alpha\cap T''\neq\varnothing$, $Y_{T'''}^\alpha\cap T'''\neq\varnothing$;

11) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$,

სადაც $T\in\{Z_2,Z_1\}$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_T^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს:, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_T^\alpha\supseteq T$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_5^\alpha\cap Z_5\neq\varnothing$ $Y_T^\alpha\cap T\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

12) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)\cup\left(Y_{T'\cup T''\cup T'''}^\alpha\times\left(T'\cup T''\cup T'''\right)\right)$

სადაც $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset T'''$, $\left(T'\cup T''\right)\setminus T'''\neq\varnothing$, $T'''\setminus\left(T'\cup T''\right)\neq\varnothing$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha,Y_{T'\cup T''\cup T'''}^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_T^\alpha\cup Y_{T''}^\alpha\cup Y_{T'''}^\alpha\supseteq T'''$, $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$ $Y_{T'''}^\alpha\cap T'''\neq\varnothing$;

13) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$

სადაც $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$, $Y_1^\alpha\cap Z_1\neq\varnothing$;

14) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$

სადაც $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

15) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_6^\alpha \times Z_6\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$,

სადაც $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$ $Y_1^\alpha \cap Z_1 \neq \varnothing$;

16) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_6^\alpha \times Z_6\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_3^\alpha \times Z_3\right) \cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$

სადაც $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_2^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_6^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$;

ლემა 3.1. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_1)\right|$-ის სიმძლავრე გამოითვლება ფორმულით $\left|I^*(Q_1)\right| = 8$.

ლემა 3.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_2)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}
\left|I^*(Q_2)\right| = {} & \left(2^{|\breve{D}\setminus Z_7|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \left(2^{|\breve{D}\setminus Z_6|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \left(2^{|\breve{D}\setminus Z_5|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \\
& + \left(2^{|\breve{D}\setminus Z_4|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \left(2^{|\breve{D}\setminus Z_3|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \left(2^{|\breve{D}\setminus Z_2|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \\
& + \left(2^{|\breve{D}\setminus Z_1|}-1\right)\cdot 2^{|X\setminus\breve{D}|} + \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_6|} + \left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_5|} + \\
& + \left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_4|} + \left(2^{|Z_3\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_3|} + \left(2^{|Z_2\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_2|} + \\
& + \left(2^{|Z_1\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_1|} + \left(2^{|Z_4\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_4|} + \left(2^{|Z_2\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_2|} + \\
& + \left(2^{|Z_1\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_1|} + \left(2^{|Z_4\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_4|} + \left(2^{|Z_3\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_3|} + \\
& + \left(2^{|Z_2\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_2|} + \left(2^{|Z_1\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_1|} + \left(2^{|Z_2\setminus Z_4|}-1\right)\cdot 2^{|X\setminus Z_2|} + \\
& + \left(2^{|Z_1\setminus Z_4|}-1\right)\cdot 2^{|X\setminus Z_1|} + \left(2^{|Z_1\setminus Z_3|}-1\right)\cdot 2^{|X\setminus Z_1|}
\end{aligned}$$

ლემა 3.3. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$.თუ X არის სასრულო სიმრავლეა მაშინ $\left|I^*(Q_3)\right|$-ის სიმძლავრე გამოითვლება ფორმულით:

$$
\begin{aligned}
\left|I^{*}(Q_3)\right| = {} & \left(2^{|Z_1\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_5|}-2^{|\breve{D}\setminus Z_5|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_6|}-2^{|\breve{D}\setminus Z_6|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_4|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_4|}+\\
& +\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_3|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot 3^{|X\setminus Z_1|}+\\
& +\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\\
& +\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_1\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_2\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_2\setminus Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_1\setminus Z_3|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}
\end{aligned}
$$

ლემა 3.4. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^{*}(Q_4)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|I^{*}(Q_4)\right| = {} & \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\\
& \cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\\
& \cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
& +\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\\
& \cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\\
& \cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
& +\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_2|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\\
& \cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\\
& \cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\times 4^{|X\setminus Z_2|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}\\
& +\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot\\
& \cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\times\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\times 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\\
& \cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

ლემა 3.5. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_5)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით:

$$\begin{aligned}\left|I^*(Q_5)\right| &= \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\end{aligned}$$

ლემა 3.6. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_6)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით:

$$\begin{aligned}\left|I^*(Q_6)\right| &= \left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\ &+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\ &+\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}\end{aligned}$$

ლემა 3.7. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_7)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*(Q_7)\right| &= \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_6|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus Z_1|}\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\ &+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\end{aligned}$$

ლემა 3.8. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_8)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით:

$$\left|I^*(Q_8)\right| = \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot \\ \cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|} \\ \cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}$$

ლემა 3.9. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_9)\right|$-ის სიმძლავრე გამოითვლება ფორმულით:

$$\left|I^*(Q_9)\right| = \left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot \\ \cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|};$$

ლემა 3.10. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{10})\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{10})\right| = \left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot 5^{|X\setminus \breve{D}|}+ \\ +\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}+ \\ +\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}+ \\ +\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot 5^{|X\setminus Z_2|}+ \\ +\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot 5^{|X\setminus Z_1|}+ \\ +\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}$$

ლემა 3.11. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{11})\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{11})\right| = \left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus \breve{D}|} \\ +\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}$$

ლემა 3.12. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{12})\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{12})\right| = \left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 6^{|X\setminus Z_1|}+ \\ +\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}+ \\ +\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}+ \\ +\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}$$

ლემა 3.13. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_{13})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{13})\right|=\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot \\ \cdot\left(3^{|Z_4\setminus Z_2|}-2^{|Z_4\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

ლემა 3.14. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_{14})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{14})\right|=\left(2^{|Z_5\setminus Z_4|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

ლემა3.15. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_{15})\right|$ -ი სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{15})\right|=\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot \\ \cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

ლემა3.16.ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*(Q_{16})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით:

$$\left|I^*(Q_{16})\right|=\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot \\ \cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}$$

თეორემა 3.2. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$.თუ X არის სასრულო სიმრავლე და $I(D)$ არის ყველა იდემპოტენტების სიმრავლე მაშინ მათი რაოდენობის გამოსათვლელ ფორმულას ექნება შემდეგი სახე $\left|I(D)\right|=\sum_{i=1}^{16}\left|\mathrm{I}(Q_i)\right|$

4. $\Sigma_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული $B_X(D)$ ნახევარჯგუფის იდეპოტენტური ელემენტები, როცა $Z_7=\varnothing$.

მოცემულ პარაგრაფში განხილულია $\Sigma_3(X,8)$ კლასით განსაზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფები. განხილულია ის შემთხვევა როცა X სასრულო სიმრავლეა და $Z_7=\varnothing$. გამოყვანილია ფორმულები, რომელთა საშუალებითაც დათვლილია მოცემული ნახევარჯგუფის იდემპოტენტური ელემენტების რაოდენობა.

ლემა 4.1 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*\left(Q_1\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით $\left|I^*\left(Q_1\right)\right|=1$.

ლემა 4.2.ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*\left(Q_2\right)\right|$ -ს სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_2\right)\right|=&\left(2^{|\breve{D}|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|Z_6|}-1\right)\cdot 2^{|X\setminus Z_6|}+\left(2^{|Z_5|}-1\right)\cdot 2^{|X\setminus Z_5|}+\left(2^{|Z_4|}-1\right)\cdot 2^{|X\setminus Z_4|}+\\&+\left(2^{|Z_3|}-1\right)\cdot 2^{|X\setminus Z_3|}+\left(2^{|Z_2|}-1\right)\cdot 2^{|X\setminus Z_2|}+\left(2^{|Z_1|}-1\right)\cdot 2^{|X\setminus Z_1|}\end{aligned}$$

ლემა 4.3 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*\left(Q_3\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_3\right)\right|=&\left(2^{|Z_1|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\&+\left(2^{|Z_3|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_5|}-2^{|\breve{D}\setminus Z_5|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_6|}-2^{|\breve{D}\setminus Z_6|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\&+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_4|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_2|}+\\&+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_4|}+\\&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_3|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_2|}+\\&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\\&+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot 3^{|X\setminus Z_1|}\end{aligned}$$

ლემა 4.4 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*\left(Q_4\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_4\right)\right|=&\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\\&\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\\&\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\end{aligned}$$

$$
\begin{aligned}
&\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_3|}-1\right)\cdot\\
&\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\\
&\cdot 4^{|X\setminus Z_2|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\\
&\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_2|}+\\
&+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}
\end{aligned}
$$

ლემა 4.5 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_4\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|I^*\left(Q_5\right)\right|=&\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 4.6 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_6\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|I^*\left(Q_6\right)\right|=&\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}\\
&+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 4. 7. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრულო სიმრავლემაშინ $\left|I^*\left(Q_7\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|I^*\left(Q_7\right)\right|=&\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_6|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}\\
&+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus Z_1|}\\
&+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}\\
&+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}\\
&+\left(2^{|Z_4|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 4.8 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_8\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_8\right)\right| = &\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\\ &\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}\\ &+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\\ &\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}\end{aligned}$$

ლემა 4.9 ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_9\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_9\right)\right| = &\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\\ &\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|};\end{aligned}$$

ლემა 4.10. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_{10}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_{10}\right)\right| = &\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\ &+\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\ &+\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\ &+\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot 5^{|X\setminus Z_2|}+\\ &+\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot 5^{|X\setminus Z_1|}+\\ &+\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}\end{aligned}$$

ლემა 4.11.ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_{11}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_{11}\right)\right| = &\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus \breve{D}|}\\ &+\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}\end{aligned}$$

ლემა 4.12. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$.თუ X არის სასრულო სიმრავლე მაშინ $\left|I^*\left(Q_{12}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|I^*\left(Q_{12}\right)\right| = &\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 6^{|X\setminus Z_1|}+\\ &+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}+\\ &+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus \breve{D}|}\end{aligned}$$

ლემა 4.13. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{13})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{13})\right| = \left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot \cdot\left(3^{|Z_4\setminus Z_2|}-2^{|Z_4\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus \breve{D}|}$$

ლემა 4.14. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{14})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{14})\right| = \left(2^{|Z_5\setminus Z_4|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus \breve{D}|};$$

ლემა 4.15 ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{15})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{15})\right| = \left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot \cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus \breve{D}|};$$

ლემა 4.16 ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$.თუ X არის სასრული სიმრავლე მაშინ $\left|I^*(Q_{16})\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|I^*(Q_{16})\right| = \left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot \cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus \breve{D}|}$$

თეორემა 4.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$. თუ X არის სასრული სიმრავლე და $I(D)$ არის ყველა იდემპოტენტების სიმრავლე მაშინ მათი რაოდენობის გამოსათვლელ ფორმულას ექნება შემდეგი სახე $\left|I(D)\right| = \sum_{i=1}^{16}\left|\mathrm{I}(Q_i)\right|$

5. $\sum_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული ბინარულ მიმართებათა $B_X(D)$ ნახევარჯგუფის მაქსიმალური ქვეჯგუფები

ამ პარაგრაფში მოცემულია $\sum_3(X,8)$ კლასის ნახევარმესერებით განსაზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფის მაქსიმალური ქვეჯგუფების სრული აღწერა.

ლემა 5.1. იმ ნახევარმესერების ავტომორფიზმთა რიცხვა, რომლების განსაზღვრული არიან ნახაზი 3.4 –ის 1), 2), 3), 4), 5), 12), 13), 14) და 16) დიაგრამებით ტოლია 1-ის , იმ ნახევარმესერების

ავტომორფოზმთა რიცხვი, რომლებიც განსაზღვრული არიან ნახაზი 3.4-ის 6), 7), 8), 9), 10) და 11)დიაგრამებით ტოლია 2 -ის და იმ ნახევარმესერების ავტომორფოზმთა რიცხვი, რომლებიც განსაზღვრული არიან ნახაზი 3.4-ის 15) დიაგრამით ტოლია 4-ის.

თეორემა 5.1. $B_X(D)$ ნახევარჯგუფის ნებისმიერი ε ბინარული მიმართებისათვის, $B_X(D)$ ნახევარჯგუფის $G_X(D,\varepsilon)$ ქვეჯგუფი წარმოადგენს ჯგუფს,რომლის რიგი არის ერთის, ორის ან ოთხის ტოლი.

6. $\Sigma_3(X,8)$ **კლასის ნახევარმესერებით განსაზღვრული** $\mathrm{B}_{\mathrm{X}}(D)$ **ბინარულ მიმართებათა სრული ნახევარჯგუფის რეგულარული ელემენტები როცა** $Z_7 \neq \varnothing$ **.** მოცემულ პარაგრაფში განხილულია $\Sigma_3(X,8)$,კლასით გან-საზღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფები და აღწერილია მათი რეგულარული ელემენტები განხილულია ის შემთხვევა როცა X სასრულო სიმრავლეა და $Z_7 \neq \varnothing$. გამოყვანილია ფორმულები, რომელთასაშუალებითაც დათვლილია მოცემული ნახევარჯგუფის რეგულარული ელემენტების რაოდენობა.

თეორემა 6.1. ვთქვათ $\Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. $B_X(D)$ ნახევარჯგუფის α ბინარული მიმართება არის მოცემული ნახევარჯგუფის რეგულარული ელემენტი მაშინ და მხოლოდ მაშინ, როცა არსებობს φ სრული $\alpha-$იზომორფიზმი $V(D,\alpha)$ ნახევარმესერისა D ნახევარ-მესერის რომელიღაც D' ქვენახევარმესერზე, რომელიც აკმაყოფილებს ქვემოთ მოცემული პირობებიდან ერთ-ერთს:

1) $\alpha = X \times T$, სადაც $T \in D$;

2) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right)$, სადაც $T, T' \in D$, $T \subset T'$, $Y_T^\alpha, Y_{T'}^\alpha \notin \{\varnothing\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha \supseteq \varphi(T)$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$;

3) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right)$, სადაც $T, T', T'' \in D$, $T \subset T' \subset T''$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha \notin \{\varnothing\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$;

4) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T'''}^\alpha \times T'''\right)$, სადაც $T, T', T'', T''' \in D$, $T \subset T' \subset T'' \subset T'''$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$, და აკმაყოფილებს შემდეგ

პირობებს: $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_{T'''}^\alpha \cap \varphi(T''') \neq \varnothing$;

5) $\alpha = (Y_7^\alpha \times Z_7) \cup (Y_T^\alpha \times T) \cup (Y_{T'}^\alpha \times T') \cup (Y_{T''}^\alpha \times T'') \cup (Y_0^\alpha \times \breve{D})$, სადაც $Y_7^\alpha, Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_0^\alpha \notin \{\varnothing\}$, და აკმაყოფილებს შემდეგ პირობებს $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_T^\alpha \supseteq \varphi(T)$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_T^\alpha \cap \varphi(T) \neq \varnothing$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

6) $\alpha = (Y_T^\alpha \times T) \cup (Y_{T'}^\alpha \times T') \cup (Y_{T''}^\alpha \times T'') \cup (Y_{T' \cup T''}^\alpha \times (T' \cup T''))$, სადაც $T, T', T'' \in D$, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T''}^\alpha \supseteq T''$, $Y_{T'}^\alpha \cap T' \neq \varnothing$, $Y_{T''}^\alpha \cap T'' \neq \varnothing$;

7) $\alpha = (Y_T^\alpha \times T) \cup (Y_{T'}^\alpha \times T') \cup (Y_{T''}^\alpha \times T'') \cup (Y_{T'''}^\alpha \times T''') \cup (Y_{T'' \cup T'''}^\alpha \times (T'' \cup T'''))$, სადაც $T \subset T' \subset T''$, $T \subset T' \subset T'''$, $T'' \setminus T''' \neq \varnothing$, $T''' \setminus T'' \neq \varnothing$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T'''}^\alpha \supseteq \varphi(T''')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_{T'''}^\alpha \cap \varphi(T''') \neq \varnothing$.

8) $\alpha = (Y_7^\alpha \times Z_7) \cup (Y_T^\alpha \times T) \cup (Y_4^\alpha \times Z_4) \cup (Y_2^\alpha \times Z_2) \cup (Y_1^\alpha \times Z_1) \cup (Y_0^\alpha \times \breve{D})$, სადაც $T \in \{Z_6, Z_5\}$, $Y_7^\alpha, Y_T^\alpha, Y_4^\alpha, Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_T^\alpha \supseteq \varphi(T)$,, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \supseteq Z_4$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$, $Y_T^\alpha \cap \varphi(T) \neq \varnothing$,, $Y_4^\alpha \cap Z_4 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$, $Y_1^\alpha \cap Z_1 \neq \varnothing$

9) $\alpha = (Y_7^\alpha \times Z_7) \cup (Y_5^\alpha \times Z_5) \cup (Y_4^\alpha \times Z_4) \cup (Y_3^\alpha \times Z_3) \cup (Y_1^\alpha \times Z_1) \cup (Y_0^\alpha \times \breve{D})$, სადაც $Y_7^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \supseteq Z_4$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_4^\alpha \cap Z_4 \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

10) $\alpha = (Y_T^\alpha \times T) \cup (Y_{T'}^\alpha \times T') \cup (Y_{T''}^\alpha \times T'') \cup (Y_{T' \cup T''}^\alpha \times (T' \cup T'')) \cup (Y_{T'''}^\alpha \times T''')$, სადაც $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T' \cup T'' \subset T'''$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq T'$, $Y_T^\alpha \cup Y_{T''}^\alpha \supseteq T''$, $Y_{T'}^\alpha \cap T' \neq \varnothing$, $Y_{T''}^\alpha \cap T'' \neq \varnothing$, $Y_{T'''}^\alpha \cap T''' \neq \varnothing$;

11) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, სადაც $T\in\{Z_2,Z_1\}$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_T^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_T^\alpha\supseteq\varphi(T)$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_5^\alpha\cap Z_5\neq\varnothing$ $Y_T^\alpha\cap T\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

12) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)\cup\left(Y_{T'\cup T''\cup T'''}^\alpha\times\left(T'\cup T''\cup T'''\right)\right)$ სადაც $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset T'''$, $\left(T'\cup T''\right)\setminus T'''\neq\varnothing$, $T'''\setminus\left(T'\cup T''\right)\neq\varnothing$, , $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha,Y_{T'\cup T''\cup T'''}^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს:, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi(T')$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi(T'')$, $Y_T^\alpha\cup Y_{T''}^\alpha\cup Y_{T'''}^\alpha\supseteq\varphi(T''')$, $Y_{T'}^\alpha\cap\varphi(T')\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi(T'')\neq\varnothing$, $Y_{T'''}^\alpha\cap\varphi(T''')\neq\varnothing$, ;

13) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$ სადაც $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$, $Y_1^\alpha\cap Z_1\neq\varnothing$;

14) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, სადაც $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

15) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, სადაც $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_2^\alpha,Y_1^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_2^\alpha\cap Z_2\neq\varnothing$, $Y_1^\alpha\cap Z_1\neq\varnothing$;

16) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$ სადაც $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ და აკმაყოფილებს შემდეგ პირობებს: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_6^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_2^\alpha\cap Z_2\neq\varnothing$;

ლემა 6.1. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X სასრული სიმრავლეა , მაშინ $\left|R^*(Q_1)\right|$-ის სიმძლავრე გამოითვლება ფორმულით $\left|R^*(Q_1)\right| = 8$.

ლემა 6.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_2)\right|$-ის სიმძლავრე გამოითვლება ფორმულით $\left|R^*(Q_2)\right| = 23 \cdot \left(2^{\left|\breve{D}\setminus Z_7\right|} - 1\right) \cdot 2^{\left|X\setminus \breve{D}\right|}$

ლემა 6.3. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_3)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_3)\right| = {} & 31\cdot\left(2^{|Z_1\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+31\cdot\left(2^{|Z_2\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_2\right|}-2^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot \\
& \cdot 3^{\left|X\setminus\breve{D}\right|}+31\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_3\right|}-2^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+31\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot \\
& \cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+31\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_5\right|}-2^{\left|\breve{D}\setminus Z_5\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+ \\
& +31\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_6\right|}-2^{\left|\breve{D}\setminus Z_6\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+31\cdot 2^{|Z_1\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot \\
& \cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-31\cdot 2^{|Z_1\setminus Z_3|}\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}- \\
& -31\cdot 2^{|Z_1\setminus Z_4|}\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-31\cdot 2^{|Z_2\setminus Z_4|}\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot \\
& \cdot\left(3^{\left|\breve{D}\setminus Z_2\right|}-2^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-31\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_3\right|}-2^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}- \\
& -31\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-31\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot \\
& \cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}.
\end{aligned}
$$

ლემა 6.4. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_4)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_4)\right| = {} & 20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+ \\
& +20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+ \\
& +20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+ \\
& +20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+
\end{aligned}
$$

$$
\begin{aligned}
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 3^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_3|}\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

ლემა 6.5. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_5)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_5)\right|=&\,5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+5\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}
\end{aligned}
$$

ლემა 6.6. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_6)\right|$-ის სიმძლავრე გამოითვლება

ფორმულით

$$
\begin{aligned}
\left|R^*(Q_6)\right| &= 20\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+20\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\
&+20\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_4|}+20\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\
&+20\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-10\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\\
&\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-10\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\\
&\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\
&-10\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\\
&\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-10\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-10\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\\
&\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

ლემა 6.7. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_7)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_7)\right| &= 14\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+14\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_6|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+14\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_4\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&-7\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\\
&\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\\
&\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-7\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\\
&\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{||Z_1\setminus Z_2||}-2^{||Z_1\setminus Z_2||}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\\
&\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\\
&\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\\
&\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus Z_1|}\cdot\\
&\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_2\setminus Z_1|}\cdot\\
&\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}.
\end{aligned}
$$

ლემა 6.8. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7\neq\varnothing$. თუ X სასრული

სიმრავლეა მაშინ $\left|R^*\left(Q_8\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_8\right)\right| = {} & 4\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_6\right|}-2^{\left|Z_4\setminus Z_6\right|}\right)\cdot 3^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot\left(4^{\left|Z_1\setminus Z_2\right|}-3^{\left|Z_1\setminus Z_2\right|}\right)\cdot\\ & \cdot\left(4^{\left|Z_2\setminus Z_1\right|}-3^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}+4\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\\ & \cdot 3^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot\left(4^{\left|Z_1\setminus Z_2\right|}-3^{\left|Z_1\setminus Z_2\right|}\right)\cdot\left(4^{\left|Z_2\setminus Z_1\right|}-3^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}.\end{aligned}$$

ლემა 6.9. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_9\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_9\right)\right|=\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_3\cap Z_4\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot\left(3^{\left|Z_4\setminus Z_3\right|}-2^{\left|Z_4\setminus Z_3\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}$$

ლემა 6.10. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{10}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}R^*\left(Q_{10}\right)= {} & 12\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_4\right|}-4^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\ & +12\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_6\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\ & +12\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\ & -6\cdot 2^{\left|Z_6\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 2^{\left|Z_3\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\ & -6\cdot 2^{\left|Z_3\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot 2^{\left|Z_6\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\ & -6\cdot 2^{\left|Z_4\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 2^{\left|Z_3\setminus Z_1\right|}\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\ & -6\cdot 2^{\left|Z_3\setminus Z_1\right|}\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot 2^{\left|Z_4\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}.\end{aligned}$$

ლემა 6.11. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{11}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{11}\right)\right|= {} & 4\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot\left(5^{\left|Z_2\setminus Z_4\right|}-4^{\left|Z_2\setminus Z_4\right|}\right)\cdot\left(6^{\left|\breve{D}\setminus Z_2\right|}-5^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}+\\ & +4\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot\left(5^{\left|Z_1\setminus Z_4\right|}-4^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(6^{\left|\breve{D}\setminus Z_1\right|}-5^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}.\end{aligned}$$

ლემა 6.12. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7\neq\varnothing$. თუ X სასრული

სიმრავლეა მაშინ $\left|R^*\left(Q_{12}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}R^*\left(Q_{12}\right)=4\cdot&\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot 6^{\left|X\setminus Z_1\right|}+\\&+4\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}+\\&+4\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}+\\&-4\cdot 2^{\left|Z_4\setminus\left(Z_6\cup Z_3\right)\right|}\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus\breve{D}\right|}\end{aligned}$$

ლემა 6.13. ვთქვათ $D\in\Sigma_3\left(X,8\right)$და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{13}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{13}\right)\right|=&\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_3\cap Z_2\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_4\setminus Z_3\right|}-2^{\left|Z_4\setminus Z_3\right|}\right)\cdot\\&\cdot\left(3^{\left|Z_3\setminus Z_2\right|}-2^{\left|Z_3\setminus Z_2\right|}\right)\cdot\left(4^{\left|Z_2\setminus Z_1\right|}-3^{\left|Z_2\setminus Z_1\right|}\right)\cdot 7^{\left|X\setminus\breve{D}\right|};\end{aligned}$$

ლემა 6.14. ვთქვათ $D\in\Sigma_3\left(X,8\right)$და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{14}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_{14}\right)\right|=\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot\left(7^{\left|\breve{D}\setminus Z_1\right|}-6^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 7^{\left|X\setminus\breve{D}\right|};$$

ლემა 6.15. ვთქვათ $D\in\Sigma_3\left(X,8\right)$და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{15}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{15}\right)\right|=2\cdot&\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot 4^{\left|\left(Z_1\cap Z_2\right)\setminus Z_4\right|}\cdot\\&\cdot\left(5^{\left|Z_2\setminus Z_1\right|}-4^{\left|Z_2\setminus Z_1\right|}\right)\cdot\left(5^{\left|Z_1\setminus Z_2\right|}-4^{\left|Z_1\setminus Z_2\right|}\right)\cdot 7^{\left|X\setminus\breve{D}\right|};\end{aligned}$$

ლემა 6.16. ვთქვათ $D\in\Sigma_3\left(X,8\right)$და $Z_7\neq\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{16}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{16}\right)\right|=2\cdot&\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 2^{\left|\left(Z_3\cap Z_2\right)\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\\&\cdot\left(3^{\left|Z_3\setminus Z_2\right|}-2^{\left|Z_3\setminus Z_2\right|}\right)\cdot\left(5^{\left|Z_2\setminus Z_1\right|}-4^{\left|Z_2\setminus Z_1\right|}\right)\cdot 8^{\left|X\setminus\breve{D}\right|}\end{aligned}$$

თეორემა 6.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 \neq \varnothing$.თუ X სასრულო სიმრავლეა და R_D სიმბოლოთი ავღნიშნავთ $B_X(D)$ ნახევარჯგუფის ყველა რეგულარულ ელემეტთა სიმრავლეს მაშინ $|R_D| = \sum_{i=1}^{16} |R(Q_i)|$.

7. $\Sigma_3(X,8)$კლასის ნახევარმესერებით განსაზღვრული $B_X(D)$ ბინარულ მიმართებათა სრული ნახევარჯგუფის რეგულარული ელემენტები როცა $Z_7 = \varnothing$. მოცემულ პარაგრაფში განხილულია $\Sigma_3(X,8)$,კლასით განსაზ-ღვრული ბინარულ მიმართებათა სრული ნახევარჯგუფები და აღწერილია მათი რეგულარული ელემენტები განხილულია ის შემთხვევა როცა X სასრულო სიმრავლეა და $Z_7 = \varnothing$. გამოყვანილია ფორმულები, რომელთა საშუალებითაც დათვლილია მოცემული ნახევარჯგუფის რეგულარული ელემენტების რაოდენობა.

ლემა 7.1. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$. თუ X სასრული სიმრავლეა მაშინ $|R^*(Q_1)|$-ის სიმძლავრე გამოითვლება ფორმულით $|R^*(Q_1)| = 1$.

ლემა 7.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$. თუ X სასრული სიმრავლეა მაშინ $|R^*(Q_2)|$-ის სიმძლავრე გამოითვლებ აფორმულით
$$|R^*(Q_2)| = 7 \cdot \left(2^{|\breve{D}|} - 1\right) \cdot 2^{|X \setminus \breve{D}|}$$

ლემა 7.3. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$. თუ X სასრული სიმრავლეა მაშინ $|R^*(Q_3)|$-ის სიმძლავრე გამოითვლება ფორმულით
$$\begin{aligned}|R^*(Q_3)| = {} & 16 \cdot \left(2^{|Z_1|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_1|} - 2^{|\breve{D} \setminus Z_1|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\ & + 16 \cdot \left(2^{|Z_2|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_2|} - 2^{|\breve{D} \setminus Z_2|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\ & + 16 \cdot \left(2^{|Z_3|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_3|} - 2^{|\breve{D} \setminus Z_3|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\ & + 16 \cdot \left(2^{|Z_4|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_4|} - 2^{|\breve{D} \setminus Z_4|}\right) \cdot 3^{|X \setminus \breve{D}|} +\end{aligned}$$

$$
\begin{aligned}
&+16\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_5\right|}-2^{\left|\breve{D}\setminus Z_5\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\\
&+16\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_6\right|}-2^{\left|\breve{D}\setminus Z_6\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\\
&+16\cdot 2^{|Z_1\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_1\setminus Z_3|}\cdot\left(2^{|Z_3|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_1\setminus Z_4|}\cdot\left(2^{|Z_4|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_2\setminus Z_4|}\cdot\left(2^{|Z_4|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_2\right|}-2^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_3\right|}-2^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}-\\
&-16\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 7.4. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_4)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_4)\right| &= 15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_3\right|}-3^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
&+15\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-
\end{aligned}
$$

$$
\begin{aligned}
&-15\cdot 3^{\left|Z_6\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_2\setminus Z_4\right|}\cdot\left(3^{\left|Z_4\setminus Z_6\right|}-2^{\left|Z_4\setminus Z_6\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 3^{\left|Z_6\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_4\right|}\cdot\left(3^{\left|Z_4\setminus Z_6\right|}-2^{\left|Z_4\setminus Z_6\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 3^{\left|Z_4\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_2\setminus Z_2\right|}\cdot\left(3^{\left|Z_2\setminus Z_4\right|}-2^{\left|Z_2\setminus Z_4\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_4\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_1\right|}\cdot\left(3^{\left|Z_1\setminus Z_4\right|}-2^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_5\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_2\setminus Z_4\right|}\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_5\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_4\right|}\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_5\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_3\right|}\cdot\left(3^{\left|Z_3\setminus Z_5\right|}-2^{\left|Z_3\setminus Z_5\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_4\setminus Z_5\right|}\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_2\setminus Z_2\right|}\cdot\left(3^{\left|Z_2\setminus Z_4\right|}-2^{\left|Z_2\setminus Z_4\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_4\setminus Z_5\right|}\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_1\right|}\cdot\left(3^{\left|Z_1\setminus Z_4\right|}-2^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-15\cdot 2^{\left|Z_3\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 3^{\left|Z_1\setminus Z_1\right|}\cdot\left(3^{\left|Z_1\setminus Z_3\right|}-2^{\left|Z_1\setminus Z_3\right|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 7.5. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_5\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*\left(Q_5\right)\right|=\;&5\cdot\left(2^{\left|Z_6\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_6\right|}-2^{\left|Z_4\setminus Z_6\right|}\right)\cdot\left(4^{\left|Z_2\setminus Z_4\right|}-3^{\left|Z_2\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+5\cdot\left(2^{\left|Z_6\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_6\right|}-2^{\left|Z_4\setminus Z_6\right|}\right)\cdot\left(4^{\left|Z_1\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+5\cdot\left(2^{\left|Z_5\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\left(4^{\left|Z_2\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+5\cdot\left(2^{\left|Z_5\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\left(4^{\left|Z_1\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+5\cdot\left(2^{\left|Z_5\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_5\right|}-2^{\left|Z_3\setminus Z_5\right|}\right)\cdot\left(4^{\left|Z_1\setminus Z_3\right|}-3^{\left|Z_1\setminus Z_3\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

ლემა 7.6. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_6\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*\left(Q_6\right)\right|=\;&10\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot\left(2^{\left|Z_1\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+20\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_6\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}+\\
&+10\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot 4^{\left|X\setminus Z_4\right|}+20\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}+\\
&+10\cdot\left(2^{\left|Z_2\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-\\
&-5\cdot 2^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot 2^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}-\\
&-5\cdot 2^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 2^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-
\end{aligned}
$$

$$
\begin{aligned}
&-5\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\
&-5\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\
&-5\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\
&-5\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\
&-5\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus \breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_3\setminus \breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus \breve{D}|}.
\end{aligned}
$$

ლემა 7.7. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_6)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$
\begin{aligned}
\left|R^*(Q_7)\right| &= 10\cdot\left(2^{|Z_4|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\
&+10\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_6|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\
&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\
&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}+\\
&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_4\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 5^{|X\setminus \breve{D}|}+
\end{aligned}
$$

$$
\begin{aligned}
&-5\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}-
\end{aligned}
$$

$$
\begin{aligned}
&-5\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{\|Z_1\setminus Z_2\|}-2^{\|Z_1\setminus Z_2\|}\right)\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot 3^{|Z_2\setminus \breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus \breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_3\setminus \breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus \breve{D}|}-\\
&-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 3^{|Z_3\setminus \breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus \breve{D}|}
\end{aligned}
$$

ლემა 7.8. ვთქვათ $D\in\Sigma_3(X,8)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*(Q_8)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_8\right)\right| = 4\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot$$
$$\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+4\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot$$
$$\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.$$

ლემა 7.9. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_9\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_9\right)\right| = \left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

ლემა 7.10. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{10}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$R^*\left(Q_{10}\right) = 10\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+10\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+10\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$-5\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}.$$

ლემა 7.11. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{11}\right)\right|$-ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_{11}\right)\right| = 4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.$$

ლემა 7.12. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული

სიმრავლეა მაშინ $\left|R^*\left(Q_{12}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}R^*\left(Q_{12}\right) = {} & 3\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot 6^{\left|X\setminus Z_1\right|}+\\ & +3\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus \breve{D}\right|}+\\ & +3\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus \breve{D}\right|}+\\ & -3\cdot 2^{\left|Z_4\setminus\left(Z_6\cup Z_3\right)\right|}\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 6^{\left|X\setminus \breve{D}\right|}\end{aligned}$$

ლემა 7.13. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{13}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{13}\right)\right| = {} & \left(2^{\left|Z_5\right|}-1\right)\cdot 2^{\left|\left(Z_3\cap Z_2\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_4\setminus Z_3\right|}-2^{\left|Z_4\setminus Z_3\right|}\right)\cdot\\ & \cdot\left(3^{\left|Z_3\setminus Z_2\right|}-2^{\left|Z_3\setminus Z_2\right|}\right)\cdot\left(4^{\left|Z_2\setminus Z_1\right|}-3^{\left|Z_2\setminus Z_1\right|}\right)\cdot 7^{\left|X\setminus \breve{D}\right|};\end{aligned}$$

ლემა 7.14. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{14}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\left|R^*\left(Q_{14}\right)\right| = \left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot\left(7^{\left|\breve{D}\setminus Z_1\right|}-6^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 7^{\left|X\setminus \breve{D}\right|};$$

ლემა 7.15. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმრავლეა მაშინ $\left|R^*\left(Q_{15}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{15}\right)\right| = {} & 2\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot 4^{\left|\left(Z_1\cap Z_2\right)\setminus Z_4\right|}\cdot\\ & \cdot\left(5^{\left|Z_2\setminus Z_1\right|}-4^{\left|Z_2\setminus Z_1\right|}\right)\cdot\left(5^{\left|Z_1\setminus Z_2\right|}-4^{\left|Z_1\setminus Z_2\right|}\right)\cdot 7^{\left|X\setminus \breve{D}\right|};\end{aligned}$$

ლემა 7.16. ვთქვათ $D\in\Sigma_3\left(X,8\right)$ და $Z_7=\varnothing$. თუ X სასრული სიმ-რავლეა მაშინ $\left|R^*\left(Q_{16}\right)\right|$ -ის სიმძლავრე გამოითვლება ფორმულით

$$\begin{aligned}\left|R^*\left(Q_{16}\right)\right| = {} & 2\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 2^{\left|\left(Z_3\cap Z_2\right)\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot\\ & \cdot\left(3^{\left|Z_3\setminus Z_2\right|}-2^{\left|Z_3\setminus Z_2\right|}\right)\cdot\left(5^{\left|Z_2\setminus Z_1\right|}-4^{\left|Z_2\setminus Z_1\right|}\right)\cdot 8^{\left|X\setminus \breve{D}\right|}.\end{aligned}$$

თეორემა 7.2. ვთქვათ $D \in \Sigma_3(X,8)$ და $Z_7 = \varnothing$.თუ X სასრულო სიმრავლეა და R_D არის $B_X(D)$ ნახევარჯგუფის ყველა რეგულარულ ელემენტთა სიმრავლე მაშინ $|R_D| = \sum_{i=1}^{16} R(Q_i)$.

ნაშრომის შედეგების აპრობაცია

ნაშრომში მიღებული ძირითადი შედეგები გამოქვეყნებულია 9 საერთაშორისო ჟურნალში. ნაშრომში მიღებული შედეგები მოხსენებული იქნა საერთაშორისო კონფერენციებზე და გენერალურ სემინარებზე:

1. Of the international conference “MODERN ALGEBRA AND ITS APPLICATION”- ბათუმი, 2010 წ., $XI-$Subsemilattice of the Semilattice of class $\Sigma_3(X,8)$.
2. Of the international conference “MODERN ALGEBRA AND ITS APPLICATION”- ბათუმი, 2011წ., Idempotent Elements of the Semigroups $B_X(D)$ defined bySemilattices of the Class $\Sigma_3(X,8)$.
3. General Seminars at Department of mathematics, HACETTEPE University, Ankara, TURKEY, 2016, ,,Complete Semigroups of Binary Relations Defined By The Semilattices of a Class $\Sigma_3(X,8)$”.
4. General Seminars at Department of mathematics, CANAKKALEOnsekizMartUniversity, Canakkale, TURKEY 2016. ,,Complete Semigroups of Binary Relations Defined By The Semilattices of a Class $\Sigma_3(X,8)$”.

ნაშრომში მიღებული შედეგები ასევე განხილულია მათემატიკის დეპარტამენტის სამეცნიერო სემინარებზე.

დანართი

სადისერტაციო ნაშრომს ბოლოში ერთვის 3 დანართი. პირველ დანართში მოცემულია მაგალითი, რომელშიც აგებულია $\Sigma_3(X,8)$ კლასის ნახევარმესერები, როცა $|X| = 4$. შემდეგ 2 დანართში მოცემულია მაგალითები, რომლებშიც სასრული X სიმრავლის შემთხვევაში აგებულია მოცემული კლასის ნახევარმესერებით

განსაზღვრული $B_X(D)$ ნახევარჯგუფების იდემპოტენტური და რეგულარული ელემენტები და ნაჩვენებია, რომ თეორიული და პრაქტიკული გამოთვლები ერთმანეთს ემთხვევა.

სადისერტაციო ნაშრომი ატარებს ძირითადად თეორიულ ხასიათს. ნაშრომში მიღებული შედეგები შეიძლება შემდგომ გამოყენებული იქნას ნახევარჯგუფთა და ნახევარმესერთა გამოკვლევებში.

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BATUMI SHOTA RUSTAVELI STATE UNIVERSITY

FACULTY OF PHYSICS-MATHEMATICS AND COMPUTER SCIENCES

DEPARTMENT OF MATHEMATICS

with the right of manuscript

Giuli Tavdgiridze

COMPLETE SEMIGROUPS OF BINARY RELATIONS DEFINED BY SEMILATTICES OF THE CLASS $\sum_3(\mathbf{X},\mathbf{8})$

This dissertation is submitted for the degree of
Academic Doctor of Mathematics
Anotation

Speciality-Mathematics

BATUMI

2017

The dissertation work has been carried out in the Department of Mathematics of Faculty of Physics-Mathematics and Computer Sciences of Batumi Shota Rustaveli State University

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The thesis will be available at the Batumi Shota Rustaveli State University Library and on the Web-page **www.bsu.edu.ge**

The secretary of the Dissertation Council,
Associate Professor **Dali Makharadze**

ACTUALITY OF THE TOPIC

The fundamental notions of the theory of binary relations were first introduced in the works of De Morgan, Pierce and Frege which dealt with mathematical logic. It should be noted that the geometrical aspect of the theory of binary relations is the graph theory. The graph theory is a part of the theory of binary relations though this theory is considered individually because it studies the properties of binary relations for special applications of the theory of binary relations.The language of binary relations is very convenient and natural for the appli-cation in mathematical linguistics, mathematical biology and in a whole number of other applied areas of mathematics.The theory of binary relations also has an important application in the au-tomata theory. In particular, each finite universal automated device can be rep-resented as a marked orientated graph called a diagram of states.

Furthermore, the theory of binary relations plays an especially important role in the part of mathematics which is called the algebra of partial mappings. Taking this fact into consideration, V.V.Wagner gave a systematic exposition of the theory of binary relations and the general theory of $n-$relations in the most suitable form for the application in the algebra of partial mappings.

Since any semigroup can be isomorphically embedded into some semi-group of binary relations, when investigating subsemigroups of a semigroup we generally study semigroups. Besides, idempotents, unilateral units, irreducible and external elements of semigroups play an exceptionally important role in the study of the abstract properties of semigroups. Difficulties encountered in studying semigroups of binary relations arise be-cause of the fact that they are not regular as a rule, which makes their investigation technically problematic. So we think it interesting to carry out a systematic study of semigroups of binary relations and their most important classes using complete $X-$semilattices of unions. which used first by one of the authors Yasha Diasamidze in his doctor dissertation. It is this new direction, we can describe it in a few words as follows.

$\Sigma(X,m)$ is the class complete $X-$semilattices of unions whose every element have the power m. Let $D_1, D_2, ..., D_n$ are any complete $X-$semilattices

of unions of the class $\Sigma(X,m)$ for which D_i and D_j are not isomorphic $(1 \le i \ne j \le n)$. $\Sigma_k(X,m)$ is the class complete $X-$semilattices of unions whose every element is isomorphic to some fixed semilattice D_k $(1 \le k \le n)$ which has the power m.

By using the complete X – semilattices, the study of complete set $B_X(D)$ of all a binary relation semigroups and spreading of the results are highly supported by Sh. Makharadze. The schientific degrees in this direction were successfully defended by Z. Avaliani, G. Partenadze, O. Givradze and N. Rokva, also the degree of mathematical academic doctors were defended by D. Ieshil and B. Albayrak in the republic of Turkey.

GOALS OF THESIS

The main goal of the dissertation work is to examine the characteristics of complete semigroups abstract features of binary relations, which are defined by complete X- semilattices of the class $\Sigma_3(X,8)$.

The research subjects are complete X – semilatteces of the class $\Sigma_3(X,8)$, as well as all complete semigroups of all binary relations defined by those semilettices, because the given semilattices, as the research shows, carry important information about complete semigroups of those binary relations, which are defined by semilattices

SCIENTIFIC NOVELTY OF THESIS

For the first time, the classes of all complete $B_X(D)$ of binary relations are discussed, which are defined by class $\Sigma_3(X,8)$ semilettices. The study of semigroup class and each element of the class is done based on the characteristics of the complete X -semilettices. The results obtained in the work gives us opportunity based on the diagram of semilettices defining that semigroup to talk about the many features of the given semigroup, namely, do they possess right elements, what kind of idempotent elements they would have, maximal subgroups, regular elements and other features.

All subsemilettices of class $\Sigma_3(X,8)$ are described and XI – subsemilettices are separated. When X is finite set, calculation formula for semilettices of class $\Sigma_3(X,8)$ quantity is shown.

Idempotent elements of semigroup $B_X(D)$ of $\Sigma_3(X,8)$ defined by each D lettices and regular elements are described. When X is finite set, calculation formula is shown. Also, maximal subgroups of semigroup $B_X(D)$ defined by semilettices of class $\Sigma_3(X,8)$ are described.

BASIC METHODS OF INVESTIGATION

As it is known, idempotent elements of semigroup, partial elements, regular elements, maximal subgroups play important role in examining of abstract features of semigroups. Therefore, a great attention is paid to those topics in studying of complete semigroups classes of binary relations.

The study of semigroups is carried out based on the features of semilettices in the dissertation work. Important part of the abstract features of complete semigroups of binary relations is mainly determined by those complete X- semilettuces, which define elements of given class. In the work, the theory of lettice, semigroup theory, set theory and general method of combinatorics are used.

THE PRACTICAL AND THEORETICAL IMPORTANCE OF THE THESIS

The dissertation is mainly theoretical. The results of the work can be used for investigating semigroups and semilattices in future.

GENERAL CHARACTERIZATION OF THESIS

1. Regular Elements and idempotents of some Complete semigroup binary relation.

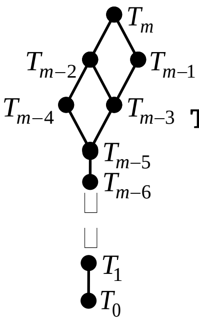

Fig.1.1

Lemma 1.1. Let $Q=\{T_0,T_1,...,T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ be $X-$semilattice of unions . Then is always an – semilattice of unions.

Theorem 1.1. Let $Q=\{T_0,T_1,...,T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ be X semilattice of unions If the $XI-$semilattices Q and $D'=\{\overline{T}_0,\overline{T}_1,...,\overline{T}_{m-3},\overline{T}_{m-2},\overline{T}_{m-1},\overline{T}_m\}$ are $\alpha-$isomorphic and $|\Omega(Q)|=m_0$, then the following equality is valid:

$$|R(D')|=m_0\cdot\left(2^{|\overline{T}_1\setminus\overline{T}_0|}-1\right)\cdot\left(3^{|\overline{T}_2\setminus\overline{T}_1|}-2^{|\overline{T}_2\setminus\overline{T}_1|}\right).$$

$$\left((m-4)^{\left|\bar{T}_{m-4}\setminus\bar{T}_{m-5}\right|}-(m-5)^{\left|\bar{T}_{m-4}\setminus\bar{T}_{m-5}\right|}\right)\cdot(m-4)^{\left|\left(\left(\bar{T}_{m-1}\cap\bar{T}_{m-4}\right)\setminus\bar{T}_{m-5}\right)\right|}\cdot$$
$$\cdot\left((m-4)^{\left|\bar{T}_{m-5}\setminus\bar{T}_{m-6}\right|}-(m-5)^{\left|\bar{T}_{m-5}\setminus\bar{T}_{m-6}\right|}\right)\cdot\left((m-3)^{\left|\bar{T}_{m-4}\setminus\bar{T}_{m-1}\right|}-(m-4)^{\left|\bar{T}_{m-4}\setminus\bar{T}_{m-1}\right|}\right)\cdot$$
$$\cdot\left((m-3)^{\left|\bar{T}_{m-3}\setminus\bar{T}_{m-4}\right|}-(m-4)^{\left|\bar{T}_{m-3}\setminus\bar{T}_{m-4}\right|}\right)\cdot\left((m-2)^{\left|\bar{T}_{m-1}\setminus\bar{T}_{m-2}\right|}-(m-3)^{\left|\bar{T}_{m-1}\setminus\bar{T}_{m-2}\right|}\right)\cdot$$
$$\cdot(m+1)^{\left|X\setminus\bar{T}_m\right|}.$$

Corollary 1.1. Let $Q=\{T_0,T_1,\ldots,T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ be $\mathrm{X}-$semilattice of unions, If $^{(\ )}(\ \ )$ is the set of all right units of the semigroup $(\ \ )$, then

$$\left|E_X^{(r)}(Q)\right|=\left(2^{\left|T_1\setminus T_0\right|}-1\right)\cdot\left(3^{\left|T_2\setminus T_1\right|}-2^{\left|T_2\setminus T_1\right|}\right)\cdots\left((m-4)^{\left|T_{m-4}\setminus T_{m-5}\right|}-(m-5)^{\left|T_{m-4}\setminus T_{m-5}\right|}\right)\cdot$$
$$\cdot(m-4)^{\left|\left(\left(T_{m-1}\cap T_{m-4}\right)\setminus T_{m-5}\right)\right|}\cdot\left((m-4)^{\left|T_{m-5}\setminus T_{m-6}\right|}-(m-5)^{\left|T_{m-5}\setminus T_{m-6}\right|}\right)\cdot\left((m-3)^{\left|T_{m-4}\setminus T_{m-1}\right|}-(m-4)^{\left|T_{m-4}\setminus T_{m-1}\right|}\right)\cdot$$
$$\cdot\left((m-3)^{\left|T_{m-3}\setminus T_{m-4}\right|}-(m-4)^{\left|T_{m-3}\setminus T_{m-4}\right|}\right)\cdot\left((m-2)^{\left|T_{m-1}\setminus T_{m-2}\right|}-(m-3)^{\left|T_{m-1}\setminus T_{m-2}\right|}\right)\cdot(m+1)^{\left|X\setminus T_m\right|}.$$

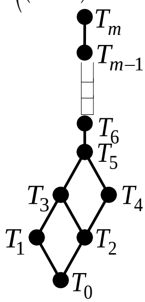

Fig.1.2

Lemma 1.2. Let $Q=\{T_0,T_1,T_2,T_3,T_4,T_5,T_6,\ldots,T_{m-1},T_m\}$ $(m\geq 6)$ be $\mathrm{X}-$semilattice of unions, Then is always an $-$ semilattice of unions

Theorem 1.2. Let $Q=\{T_0,T_1,T_2,T_3,T_4,T_5,T_6,\ldots,T_{m-1},T_m\}$ $(m\geq 6)$ be $\mathrm{X}-$semilattice of unions, If the $XI-$semilattices Q and $D'=\{\bar{T}_0,\bar{T}_1,\bar{T}_2,\ldots,\bar{T}_m\}$ are $\alpha-$isomorphic and $\left|\Omega(Q)\right|=m_0$ then the following equality is valid:

$$\left|R(D')\right|=m_0\cdot\left(3^{\left|\bar{T}_4\setminus\bar{T}_3\right|}-2^{\left|\bar{T}_4\setminus\bar{T}_3\right|}\right)\cdot\left(2^{\left|\bar{T}_1\setminus\bar{T}_4\right|}-1\right)\cdot\left(2^{\left|\bar{T}_2\setminus\bar{T}_1\right|}-1\right)\cdot\left(7^{\left|\bar{T}_6\setminus\bar{T}_5\right|}-6^{\left|\bar{T}_6\setminus\bar{T}_5\right|}\right)\cdot$$
$$\cdot\left(8^{\left|\bar{T}_7\setminus\bar{T}_6\right|}-7^{\left|\bar{T}_7\setminus\bar{T}_6\right|}\right)\cdots\left(m^{\left|\bar{T}_{m-1}\setminus\bar{T}_{m-2}\right|}-(m-1)^{\left|\bar{T}_{m-1}\setminus\bar{T}_{m-2}\right|}\right)\cdot(m+1)^{\left|X\setminus\bar{T}_m\right|}.$$

Corollary 1.2. Let $Q=\{T_0,T_1,T_2,T_3,T_4,T_5,T_6,\ldots,T_{m-1},T_m\}$ $(m\geq 6)$ be $\mathrm{X}-$semilattice of unions ,If $^{(\ )}(\ \ )$ is the set of all right units of the semigroup $(\ \ )$, then the following equality is valid:

$$\left|E_X^{(r)}(Q)\right|=\left(3^{\left|T_1\setminus T_3\right|}-2^{\left|T_4\setminus T_3\right|}\right)\cdot\left(2^{\left|T_1\setminus T_4\right|}-1\right)\cdot\left(2^{\left|T_2\setminus T_1\right|}-1\right)\cdot\left(7^{\left|T_6\setminus T_5\right|}-6^{\left|T_6\setminus T_5\right|}\right)\cdot$$
$$\cdot\left(8^{\left|T_7\setminus T_6\right|}-7^{\left|T_7\setminus T_6\right|}\right)\cdots\left(m^{\left|T_{m-1}\setminus T_{m-2}\right|}-(m-1)^{\left|T_{m-1}\setminus T_{m-2}\right|}\right)\cdot(m+1)^{\left|X\setminus T_m\right|}.$$

Lemma 1.3. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$ be X − semilattice of unions, Then is always an − semilattice of unions

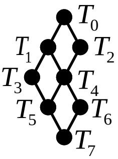

Fig. 1.3

Theorem 1.3. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$ and $|\Sigma_3(X,8)|=m_0$. If X be finite set, and the $XI-$ semilattice Q and $Q'=\{\bar{T}_7,\bar{T}_6,\bar{T}_5,\bar{T}_4,\bar{T}_3,\bar{T}_2,\bar{T}_1,\bar{T}_0\}$ are $\alpha-$ isomorphic, then the following formula is true

$$\left|R(Q')\right|=m_0\cdot\left(2^{|\bar{T}_6\setminus\bar{T}_3|}-1\right)\cdot 2^{|(\bar{T}_3\cap\bar{T}_2)\setminus\bar{T}_4|}\cdot\left(2^{|\bar{T}_5\setminus\bar{T}_6|}-1\right)\cdot$$
$$\cdot\left(3^{|\bar{T}_3\setminus\bar{T}_2|}-2^{|\bar{T}_3\setminus\bar{T}_2|}\right)\cdot\left(5^{|\bar{T}_2\setminus\bar{T}_1|}-4^{|\bar{T}_2\setminus\bar{T}_1|}\right)\cdot 8^{|X\setminus\bar{T}_0|}$$

Corollary 1.3. Let $Q=\{T_7,T_6,T_5,T_4,T_3,T_2,T_1,T_0\}\in\Sigma_3(X,8)$, If X be a finite set and $E_X^{(r)}(Q)$ be the set of all right units of the semigroup $B_X(Q)$, then the following formula is true

$$\left|E_X^{(r)}(Q)\right|=\left(2^{|T_6\setminus T_3|}-1\right)\cdot 2^{|(T_3\cap T_2)\setminus T_4|}\cdot\left(2^{|T_5\setminus T_6|}-1\right)\cdot$$
$$\cdot\left(3^{|T_3\setminus T_2|}-2^{|T_3\setminus T_2|}\right)\cdot\left(5^{|T_2\setminus T_1|}-4^{|T_2\setminus T_1|}\right)\cdot 8^{|X\setminus T_0|}$$

2. Subsemilattice of The Semilattice Class $\boldsymbol{\Sigma_3(X,8)}$ in this paragraph we will give complete classification all − subsemilattices of the semilattices of the class $\Sigma_3(X,8)$ we derive formulas by calculating the numbers of semilattices of the given class.

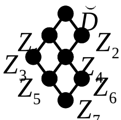

Fig. 2.1

By the simbol $\Sigma_3(X,8)$ we denote the class of all $X-$ semilattices of unions whose every element is isomorphic to $X-$ semilattice of the form $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}$, The semilattice is shown in Figure 2.1.

Lemma 2.1. Let $D\in\Sigma_3(X,8)$, $|\Sigma_3(X,8)|=s$ and $|X|\geq\delta\geq 4$. If X be a finite set, then $s=\frac{1}{2}\cdot\left(5^n-4\cdot 6^n+6\cdot 7^n-4\cdot 8^n+9^n\right)$.

Example 2.1. Let $n=4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,$ then:
$s=24,\ 840,\ 17760,\ 147000,\ 2099412,\ 27156780,\ 327284760$ and

$\left|B_X(D)\right| = 4096,\ 32768,\ 262144,\ 2097152,\ 16777216,\ 134217728,\ 1073741824.$

The number obtained show that if, for instance $|X|=10$, than the number of elements in the class of semigroups , where each element is defined by some semilattice of the class is equal to $\Sigma_3(X,8)327284760$, while the number of elements in each semigroup belonging to this class is equal to 1072741824.

Lemma 2.2. Let $\in \Sigma\ ($ $)$ and $Z_7 \neq \varnothing$. Then the following sets are all $XI-$ subsemilattices of the given semilattice D :

1) $\{Z_7\},\{Z_6\},\{Z_5\},\{Z_4\},\{Z_3\},\{Z_2\},\{Z_1\},\{\breve{D}\}$; (see diagram 1 of the figure 2.2);

2) $\{Z_7,Z_6\},\{Z_7,Z_5\},\{Z_7,Z_4\},\{Z_7,Z_3\},\{Z_7,Z_2\},\{Z_7,Z_1\},\{Z_7,\breve{D}\},$
$\{Z_6,Z_4\},\{Z_6,Z_2\},\{Z_6,Z_1\},\{Z_6,\breve{D}\},\{Z_5,Z_4\},\{Z_5,Z_3\},\{Z_5,Z_2\},$
$\{Z_5,Z_1\},\{Z_5,\breve{D}\},\ \{Z_4,Z_2\},\{Z_4,Z_1\},\{Z_4,\breve{D}\},\{Z_3,Z_1\},\{Z_3,\breve{D}\},$
$\{Z_2,\breve{D}\},\{Z_1,\breve{D}\}$,(see diagram 2 of the figure 2.2);

3) $\{Z_7,Z_6,Z_4\},\{Z_7,Z_6,Z_2\},\{Z_7,Z_6,Z_1\},\{Z_7,Z_6,\breve{D}\},\{Z_7,Z_5,Z_4\},\{Z_7,Z_5,Z_3\},$
$\{Z_7,Z_5,Z_2\},\{Z_7,Z_5,Z_1\},\{Z_7,Z_5,\breve{D}\},\{Z_7,Z_4,Z_2\},\{Z_7,Z_4,Z_1\},\{Z_7,Z_4,D\},$
$\{Z_7,Z_3,Z_1\},\{Z_7,Z_3,\breve{D}\},\{Z_7,Z_2,\breve{D}\},\{Z_7,Z_1,\breve{D}\},\{Z_6,Z_4,Z_2\},\{Z_6,Z_4,Z_1\},$
$\{Z_6,Z_4,\breve{D}\},\{Z_6,Z_2,\breve{D}\},\{Z_6,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2\},\{Z_5,Z_4,Z_1\},\{Z_5,Z_4,\breve{D}\},$
$\{Z_5,Z_3,Z_1\},\{Z_5,Z_3,\breve{D}\},\{Z_5,Z_2,\breve{D}\},\{Z_5,Z_1,\breve{D}\},\{Z_4,Z_2,\breve{D}\},\{Z_4,Z_1,\breve{D}\},$
$\{Z_3,Z_1,\breve{D}\}$;

(see diagram 3 of the figure 2.2);

4) $\{Z_7,Z_6,Z_4,Z_2\},\{Z_7,Z_6,Z_4,Z_1\},\{Z_7,Z_6,Z_4,\breve{D}\},\{Z_7,Z_6,Z_2,D\},\{Z_7,Z_6,Z_1,\breve{D}\},$
$\{Z_7,Z_5,Z_4,Z_2\},\{Z_7,Z_5,Z_4,Z_1\},\{Z_7,Z_5,Z_4,\breve{D}\}\ \{Z_7,Z_5,Z_3,Z_1\},\{Z_7,Z_5,Z_3,\breve{D}\},$
$\{Z_7,Z_5,Z_2,\breve{D}\},\{Z_7,Z_5,Z_1,\breve{D}\},\{Z_7,Z_4,Z_2,D\},\{Z_7,Z_4,Z_1,\breve{D}\},\{Z_7,Z_3,Z_1,\breve{D}\},$
$\{Z_6,Z_4,Z_2,\breve{D}\},\{Z_6,Z_4,Z_1,\breve{D}\},\{Z_5,Z_4,Z_2,\breve{D}\},\{Z_5,Z_4,Z_1,\breve{D}\},\{Z_5,Z_3,Z_1,\breve{D}\}$;
(see diagram 4 of the figure 2.2);

5) $\{Z_7,Z_6,Z_4,Z_2,\breve{D}\},\{Z_7,Z_6,Z_4,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,\breve{D}\},\{Z_7,Z_5,Z_4,Z_1,\breve{D}\},$
$\{Z_7,Z_5,Z_3,Z_1,\breve{D}\},$

(see diagram 5 of the figure 2.2);

7) $\{Z_7,Z_6,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_3,Z_1,\},\{Z_7,Z_5,Z_3,Z_2,\breve{D}\},\{Z_7,Z_5,Z_2,Z_1,\breve{D}\},$
$\{Z_7,Z_4,Z_2,Z_1,\breve{D}\},\{Z_6,Z_4,Z_2,Z_1,\breve{D}\}\{Z_5,Z_4,Z_2,Z_1,\breve{D}\}$;
(see diagram 7 of the figure 2.2);

8) $\{Z_7,Z_6,Z_4,Z_2,Z_1,\breve{D}\},\{Z_7,Z_5,Z_4,Z_2,Z_1,\breve{D}\}$;

(see diagram 8 of the figure 2.2);

9) $\{Z_7, Z_5, Z_4, Z_3, Z_1, \breve{D}\}$; (see diagram 9 of the figure 2.2);

10) $\{Z_7, Z_6, Z_5, Z_4, Z_2\}, \{Z_7, Z_6, Z_5, Z_4, Z_1\}, \{Z_7, Z_6, Z_5, Z_4, \breve{D}\}$, $\{Z_7, Z_4, Z_3, Z_1, \breve{D}\}, \{Z_7, Z_6, Z_3, Z_1, \breve{D}\}, \{Z_5, Z_4, Z_3, Z_1, \breve{D}\}$; (see diagram 10 of the figure 2.2);

11) $\{Z_7, Z_6, Z_5, Z_4, Z_2, \breve{D}\}, \{Z_7, Z_6, Z_5, Z_4, Z_1, \breve{D}\}$; (see diagram 11 of the figure2.2);

12) $\{Z_7, Z_6, Z_5, Z_4, Z_3, Z_1\}, \{Z_7, Z_6, Z_3, Z_2, Z_1, \breve{D}\}, \{Z_7, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$, $\{Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$; (see diagram 12 of the figure 2.2);

13) $\{Z_7, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$; (see diagram 13 of the figure 2.2);

14) $\{Z_7, Z_6, Z_5, Z_4, Z_3, Z_1, \breve{D}\}$; (see diagram 14 of the figure 2.2);

15) $\{Z_7, Z_6, Z_5, Z_4, Z_2, Z_1, \breve{D}\}$; (see diagram 15 of the figure 2.2);

16) $\{Z_7, Z_6, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$; (see diagram 16 of the figure 2.2);

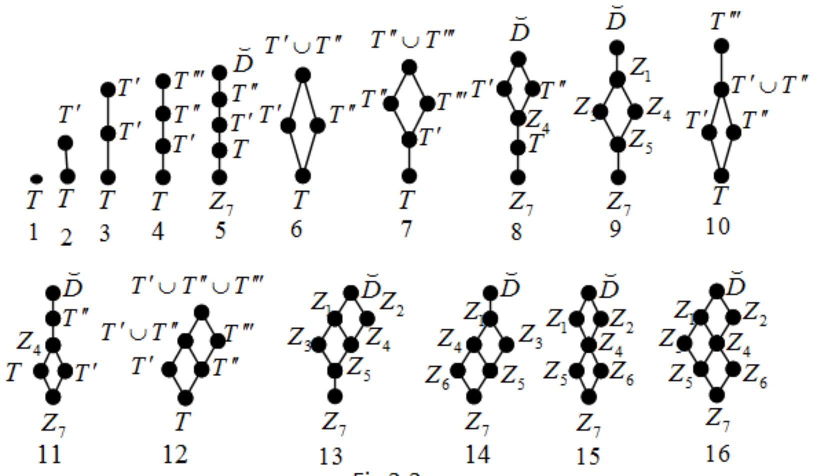

Fig 2.2

3. Idempotent Elements of the Semigroups $B_X(D)$ defined by Semilattices of the Class $\Sigma_3(X,8)$ when $Z_7 \neq \varnothing$ In the paper Complete semigroup binary relation defined by semilattices of the class $\Sigma_3(X,8)$, we give a full description of idempotent elements of given semigroup.

For the case where X is a finite set and $Z_7 \neq \varnothing$. we derive formulas by calculating the numbers of idempotent elements of the respective semigroup.

We denote the following semitattices Q_i i=$(1,2,......16)$ as follows:

1) $Q_1 = \{T\}$, where $T \in D$ (see diagram 1 of the Fig. 2.2)

2) $Q_2 = \{T, T'\}$, where $T, T' \in D$, $T \subset T'$; (see diagram 2 of the Fig. 2.2)

3) $Q_3 = \{T, T', T''\}$, where $T, T', T'' \in D$, $T \subset T' \subset T''$; (see diagram 3 of the Fig. 2.2)

4) $Q_4 = \{T, T', T'', T'''\}$, where $T, T', T'', T''' \in D$, $T \subset T' \subset T'' \subset T'''$, (see diagram 4 of the Fig. 2.2)

5) $Q_5 = \{Z_7, T, T', T'', \breve{D}\}$, where $Z_7, T, T', T'', \breve{D} \in D$, $Z_7 \subset T \subset T' \subset T'' \subset \breve{D}$, (see diagram 5 of the Fig. 2.2)

6) $Q_6 = \{T, T', T'', T' \cup T''\}$, where $T, T', T'' \in D$, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$ $T'' \setminus T' \neq \varnothing$, (see diagram 6 of the Fig. 2.2)

7) $Q_7 = \{T, T', T'', T''', T'' \cup T'''\}$, where, $T \subset T' \subset T''$, $T \subset T' \subset T'''$, $T'' \setminus T''' \neq \varnothing$ $T''' \setminus T'' \neq \varnothing$ (see diagram 7 of the Fig. 2.2)

8) $Q_8 = \{Z_7, T, Z_4, Z_2, Z_1, \breve{D}\}$, where $T \in \{Z_6, Z_5\}$ (see diagram 8 of the Fig. 2.2)

9) $Q_9 = \{Z_7, Z_5, Z_4, Z_3, Z_1, \breve{D}\}$, (see diagram 9 of the Fig. 2.2)

10) $Q_{10} = \{T, T', T'', T' \cup T'', T'''\}$, where $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T' \cup T'' \subset T'''$ (see diagram 10 of the Fig. 2.2)

11) $Q_{11} = \{Z_7, Z_6, Z_5, Z_4, T, \breve{D}\}$, where $T \in \{Z_2, Z_1\}$, (see diagram 11 of the Fig. 2.2)

12) $Q_{12} = \{T, T', T'', T' \cup T'', T''', T' \cup T'' \cup T'''\}$, where, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T'' \subset T'''$, $(T' \cup T'') \setminus T''' \neq \varnothing$, $T''' \setminus (T' \cup T'') \neq \varnothing$, (see diagram 12 of the Fig. 2.2)

13) $Q_{13} = \{Z_7, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$, (see diagram 13 of the Fig. 2.2)

Theorem 3.1. Let $\in \Sigma$ ( ), $Z_7 \neq \varnothing$ and $\alpha \in B_X(D)$. Binary relation α is an idempotent relation of the semigroup $B_X(D)$ iff binary relation α satisfies only one conditions of the following conditions:

1) $\alpha = X \times T$, where $T \in D$;

2) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)$, where $T,T'\in D$, $T\subset T'$, $Y_T^\alpha,Y_{T'}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq T$, $Y_{T'}^\alpha\cap T'\neq\varnothing$;

3) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)$, where $T,T',T''\in D$, $T\subset T'\subset T''$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq T$, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$;

4) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)$, where $T,T',T'',T'''\in D$, $T\subset T'\subset T''\subset T'''$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha\supseteq T$, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$, $Y_{T'''}^\alpha\cap T'''\neq\varnothing$;

5) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where $Z_7\subset T\subset T'\subset T''\subset\breve{D}$, $Y_7^\alpha,Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_0^\alpha\notin\{\varnothing\}$, and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_T^\alpha\supseteq T$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_T^\alpha\cap T\neq\varnothing$, $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

6) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)$, where $T,T',T''\in D$, $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$;

7) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)\cup\left(Y_{T''\cup T'''}^\alpha\times\left(T''\cup T'''\right)\right)$, where, $T\subset T'\subset T''$, $T\subset T'\subset T'''$, $T''\setminus T'''\neq\varnothing$, $T'''\setminus T''\neq\varnothing$ $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\supseteq T$ $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq T'$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T''}^\alpha\supseteq T''$, $Y_T^\alpha\cup Y_{T'}^\alpha\cup Y_{T'''}^\alpha\supseteq T'''$ $Y_{T'}^\alpha\cap T'\neq\varnothing$, $Y_{T''}^\alpha\cap T''\neq\varnothing$, $Y_{T'''}^\alpha\cap T'''\neq\varnothing$.

8) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$, where $T\in\{Z_6,Z_5\}$, $Y_7^\alpha,Y_T^\alpha,Y_4^\alpha,Y_2^\alpha,Y_1^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_T^\alpha\supseteq T$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_7^\alpha\cup Y_T^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_T^\alpha\cap T\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$ $Y_2^\alpha\cap Z_2\neq\varnothing$, $Y_1^\alpha\cap Z_1\neq\varnothing$

9) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$,where $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$,

$Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$ $Y_7^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \supseteq Z_4$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_4^\alpha \cap Z_4 \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

10) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T' \cup T''}^\alpha \times \left(T' \cup T''\right)\right) \cup \left(Y_{T'''}^\alpha \times T'''\right)$,where $T \subset T'$ $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T' \cup T'' \subset T'''$ $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq T'$, $Y_T^\alpha \cup Y_{T''}^\alpha \supseteq T''$, $Y_{T'}^\alpha \cap T' \neq \varnothing$, $Y_{T''}^\alpha \cap T'' \neq \varnothing$, $Y_{T'''}^\alpha \cap T''' \neq \varnothing$;

11) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_6^\alpha \times Z_6\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_T^\alpha \times T\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where $T \in \{Z_2, Z_1\}$, $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_T^\alpha, Y_0^\alpha \notin \{\varnothing\}$ and satisfies the conditions:, $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_T^\alpha \supseteq T$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_5^\alpha \cap Z_5 \neq \varnothing$ $Y_T^\alpha \cap T \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

12) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T' \cup T''}^\alpha \times \left(T' \cup T''\right)\right) \cup \left(Y_{T'''}^\alpha \times T'''\right) \cup \left(Y_{T' \cup T'' \cup T'''}^\alpha \times \left(T' \cup T'' \cup T'''\right)\right)$, where $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $T'' \subset T'''$, $\left(T' \cup T''\right) \setminus T''' \neq \varnothing$, $T''' \setminus \left(T' \cup T''\right) \neq \varnothing$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha, Y_{T' \cup T'' \cup T'''}^\alpha \notin \{\varnothing\}$ and satisfies the conditions:, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq T'$, $Y_T^\alpha \cup Y_{T''}^\alpha \supseteq T''$ $Y_T^\alpha \cup Y_{T''}^\alpha \cup Y_{T'''}^\alpha \supseteq T'''$, $Y_{T'}^\alpha \cap T' \neq \varnothing$, $Y_{T''}^\alpha \cap T'' \neq \varnothing$ $Y_{T'''}^\alpha \cap T''' \neq \varnothing$;

13) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_3^\alpha \times Z_3\right) \cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where $Z_5 \subset Z_3$, $Z_5 \subset Z_4$, $Z_3 \setminus Z_4 \neq \varnothing$, $Z_4 \setminus Z_3 \neq \varnothing$, $Z_4 \subset Z_2$, $Z_1 \setminus Z_2 \neq \varnothing$, $Z_2 \setminus Z_1 \neq \varnothing$, $Y_7^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_2^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$ $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \supseteq Z_4$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_4^\alpha \cap Z_4 \neq \varnothing$ $Y_1^\alpha \cap Z_1 \neq \varnothing$;

14) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_6^\alpha \times Z_6\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_3^\alpha \times Z_3\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where, $Z_6 \subset Z_4$, $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$ $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

15) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_6^\alpha \times Z_6\right) \cup \left(Y_5^\alpha \times Z_5\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$, $Y_1^\alpha \cap Z_1 \neq \varnothing$;

16) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$,
where, $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_2^\alpha, Y_1^\alpha, Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$ $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_6^\alpha\cup Y_4^\alpha\cup Y_2^\alpha\supseteq Z_2$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_2^\alpha\cap Z_2\neq\varnothing$;

Lemma 3.1. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_1\right)\right|$ my be calculated by the formula $\left|I^*\left(Q_1\right)\right|=8$.

Lemma 3.2. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_2\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_2\right)\right|=&\left(2^{|\breve{D}\setminus Z_7|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|\breve{D}\setminus Z_6|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|\breve{D}\setminus Z_5|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\\
&+\left(2^{|\breve{D}\setminus Z_4|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|\breve{D}\setminus Z_3|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|\breve{D}\setminus Z_2|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\\
&+\left(2^{|\breve{D}\setminus Z_1|}-1\right)\cdot 2^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_6|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_5|}+\\
&+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_3|}+\left(2^{|Z_2\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_2|}+\\
&+\left(2^{|Z_1\setminus Z_7|}-1\right)\cdot 2^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_4|}+\left(2^{|Z_2\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_2|}+\\
&+\left(2^{|Z_1\setminus Z_6|}-1\right)\cdot 2^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_3|}+\\
&+\left(2^{|Z_2\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_2|}+\left(2^{|Z_1\setminus Z_5|}-1\right)\cdot 2^{|X\setminus Z_1|}+\left(2^{|Z_2\setminus Z_4|}-1\right)\cdot 2^{|X\setminus Z_2|}+\\
&+\left(2^{|Z_1\setminus Z_4|}-1\right)\cdot 2^{|X\setminus Z_1|}+\left(2^{|Z_1\setminus Z_3|}-1\right)\cdot 2^{|X\setminus Z_1|}
\end{aligned}
$$

Lemma 3.3. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_3\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_3\right)\right|=&\left(2^{|Z_1\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_5|}-2^{|\breve{D}\setminus Z_5|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_6|}-2^{|\breve{D}\setminus Z_6|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_4|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_4|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_3|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot
\end{aligned}
$$

$$
\begin{aligned}
&\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\\
&\cdot 3^{|X\setminus Z_2|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_2\setminus Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot\\
&\cdot 3^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_3|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.4. Let $\in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_4)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*(Q_4)\right|=&\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\\
&\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\\
&\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\\
&\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\\
&\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_2|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\\
&\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\\
&\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\times 4^{|X\setminus Z_2|}+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}\\
&+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot\\
&\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\times\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\\
&\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.5. Let $\in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_5)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_5\right)\right| = &\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.6. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_6\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_6\right)\right| = &\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\
&+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.7. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_7\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_7\right)\right| = &\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_6|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus Z_1|}\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_4\setminus Z_6|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}\\
&+\left(2^{|Z_4\setminus Z_5|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.8. Let $\in\Sigma$ ( ) and $Z_7\neq\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_8\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_8\right)\right| = &\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\\
&\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\\
&\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 3.9. Let $\in \Sigma$ ( ) and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_9)\right|$ my be calculated by the formula

$$\left|I^*(Q_9)\right| = \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot 2^{|(Z_3 \cap Z_4) \setminus Z_5|} \cdot \left(3^{|Z_3 \setminus Z_4|} - 2^{|Z_3 \setminus Z_4|}\right) \cdot$$
$$\cdot \left(3^{|Z_4 \setminus Z_3|} - 2^{|Z_4 \setminus Z_3|}\right) \cdot \left(6^{|\breve{D} \setminus Z_1|} - 5^{|\breve{D} \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|};$$

Lemma 3.10. Let $\in \Sigma$ ( ) and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{10})\right|$ my be calculated by the formula

$$\left|I^*(Q_{10})\right| = \left(2^{|Z_6 \setminus Z_5|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(5^{|\breve{D} \setminus Z_4|} - 4^{|\breve{D} \setminus Z_4|}\right) \cdot 5^{|X \setminus \breve{D}|} +$$
$$+ \left(2^{|Z_6 \setminus Z_3|} - 1\right) \cdot \left(2^{|Z_3 \setminus Z_6|} - 1\right) \cdot \left(5^{|\breve{D} \setminus Z_1|} - 4^{|\breve{D} \setminus Z_1|}\right) \cdot 5^{|X \setminus \breve{D}|} +$$
$$+ \left(2^{|Z_4 \setminus Z_3|} - 1\right) \cdot \left(2^{|Z_3 \setminus Z_4|} - 1\right) \cdot \left(5^{|\breve{D} \setminus Z_1|} - 4^{|\breve{D} \setminus Z_1|}\right) \cdot 5^{|X \setminus \breve{D}|} +$$
$$+ \left(2^{|Z_6 \setminus Z_5|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(5^{|Z_2 \setminus Z_4|} - 4^{|Z_2 \setminus Z_4|}\right) \cdot 5^{|X \setminus Z_2|} +$$
$$+ \left(2^{|Z_6 \setminus Z_5|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(5^{|Z_1 \setminus Z_4|} - 4^{|Z_1 \setminus Z_4|}\right) \cdot 5^{|X \setminus Z_1|} +$$
$$+ \left(2^{|Z_4 \setminus Z_3|} - 1\right) \cdot \left(2^{|Z_3 \setminus Z_4|} - 1\right) \cdot \left(5^{|\breve{D} \setminus Z_1|} - 4^{|\breve{D} \setminus Z_1|}\right) \cdot 5^{|X \setminus \breve{D}|}$$

Lemma 3.11. Let $\in \Sigma$ ( ) and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{11})\right|$ my be calculated by the formula

$$\left|I^*(Q_{11})\right| = \left(2^{|Z_6 \setminus Z_5|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(5^{|Z_2 \setminus Z_4|} - 4^{|Z_2 \setminus Z_4|}\right) \cdot \left(6^{|\breve{D} \setminus Z_2|} - 5^{|\breve{D} \setminus Z_2|}\right) \cdot 6^{|X \setminus \breve{D}|}$$
$$+ \left(2^{|Z_6 \setminus Z_5|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(5^{|Z_1 \setminus Z_4|} - 4^{|Z_1 \setminus Z_4|}\right) \cdot \left(6^{|\breve{D} \setminus Z_1|} - 5^{|\breve{D} \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|}$$

Lemma 3.12. Let $\in \Sigma$ ( ) and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{12})\right|$ my be calculated by the formula

$$\left|I^*(Q_{12})\right| = \left(2^{|Z_6 \setminus Z_3|} - 1\right) \cdot \left(2^{|Z_5 \setminus Z_6|} - 1\right) \cdot \left(3^{|Z_3 \setminus Z_4|} - 2^{|Z_3 \setminus Z_4|}\right) \cdot 6^{|X \setminus Z_1|} +$$
$$+ \left(2^{|Z_3 \setminus Z_2|} - 1\right) \cdot \left(2^{|Z_6 \setminus Z_3|} - 1\right) \cdot \left(3^{|Z_2 \setminus Z_1|} - 2^{|Z_2 \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|} +$$
$$+ \left(2^{|Z_3 \setminus Z_2|} - 1\right) \cdot \left(2^{|Z_4 \setminus Z_3|} - 1\right) \cdot \left(3^{|Z_2 \setminus Z_1|} - 2^{|Z_2 \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|} +$$
$$+ \left(2^{|Z_3 \setminus Z_2|} - 1\right) \cdot \left(2^{|Z_4 \setminus Z_3|} - 1\right) \cdot \left(3^{|Z_2 \setminus Z_1|} - 2^{|Z_2 \setminus Z_1|}\right) \cdot 6^{|X \setminus \breve{D}|}$$

Lemma 3.13. Let $\in \Sigma$ ( ) and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{13})\right|$ my be calculated by the formula

$$\left|I^*(Q_{13})\right| = \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot 2^{|(Z_3 \cap Z_2) \setminus Z_5|} \cdot \left(3^{|Z_3 \setminus Z_4|} - 2^{|Z_3 \setminus Z_4|}\right) \cdot$$
$$\cdot \left(3^{|Z_4 \setminus Z_2|} - 2^{|Z_4 \setminus Z_2|}\right) \cdot \left(4^{|Z_2 \setminus Z_1|} - 3^{|Z_2 \setminus Z_1|}\right) \cdot 7^{|X \setminus \breve{D}|};$$

Lemma 3.14. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{14})\right|$ my be calculated by the formula

$$\left|I^*(Q_{14})\right| = \left(2^{|Z_5 \setminus Z_4|}-1\right)\cdot\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(3^{|Z_4 \setminus Z_3|}-2^{|Z_4 \setminus Z_3|}\right)\cdot\left(7^{|\breve{D} \setminus Z_1|}-6^{|\breve{D} \setminus Z_1|}\right)\cdot 7^{|X \setminus \breve{D}|};$$

Lemma 3.15. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{15})\right|$ my be calculated by the formula

$$\left|I^*(Q_{15})\right| = \left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot 4^{|(Z_1 \cap Z_2) \setminus Z_4|}\cdot$$
$$\cdot\left(5^{|Z_2 \setminus Z_1|}-4^{|Z_2 \setminus Z_1|}\right)\cdot\left(5^{|Z_1 \setminus Z_2|}-4^{|Z_1 \setminus Z_2|}\right)\cdot 7^{|X \setminus \breve{D}|};$$

Lemma 3.16. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{16})\right|$ my be calculated by the formula

$$\left|I^*(Q_{16})\right| = \left(2^{|Z_6 \setminus Z_3|}-1\right)\cdot 2^{|(Z_3 \cap Z_2) \setminus Z_4|}\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(3^{|Z_3 \setminus Z_2|}-2^{|Z_3 \setminus Z_2|}\right)\cdot\left(5^{|Z_2 \setminus Z_1|}-4^{|Z_2 \setminus Z_1|}\right)\cdot 8^{|X \setminus \breve{D}|}$$

Theorem 3.2. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 \neq \varnothing$. If X is a finite set, then the number $\left|I(D)\right|$ my be calculated by the formula $\left|I(D)\right| = \sum_{i=1}^{16}\left|I^*(Q_i)\right|$.

4. Idempotent Elements of the Semigroups $B_X(D)$ definedby Semilattices of the Class $\Sigma_3(X,8)$, when $Z_7 = \varnothing$ In the paper Complete semigroup binary relation defined by semilattices of the class $\Sigma_3(X,8)$, we give a full description of idempotent elements of given semigroup. For the case where X is a finite set and $Z_7 = \varnothing$. we derive formulas by calculating the numbers of idempotent elements of the respective semigroup.

Lemma 4.1. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_1)\right|$ my be calculated by the formula $\left|I^*(Q_1)\right| = 1$.

Lemma 4.2. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_2)\right|$ my be calculated by the formula

$$\left|I^*(Q_2)\right| = \left(2^{|\breve{D}|}-1\right)\cdot 2^{|X \setminus \breve{D}|} + \left(2^{|Z_6|}-1\right)\cdot 2^{|X \setminus Z_6|} + \left(2^{|Z_5|}-1\right)\cdot 2^{|X \setminus Z_5|} +$$
$$+\left(2^{|Z_4|}-1\right)\cdot 2^{|X \setminus Z_4|} + \left(2^{|Z_3|}-1\right)\cdot 2^{|X \setminus Z_3|} + \left(2^{|Z_2|}-1\right)\cdot 2^{|X \setminus Z_2|} + \left(2^{|Z_1|}-1\right)\cdot 2^{|X \setminus Z_1|}$$

Lemma 4.3. Let $\quad \in \Sigma\ (\qquad)$ and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_3)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_3\right)\right| = {} & \left(2^{|Z_1|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_1\right|}-2^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_2|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_2\right|}-2^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\\
& +\left(2^{|Z_3|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_3\right|}-2^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_4\right|}-2^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\\
& +\left(2^{|Z_5|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_5\right|}-2^{\left|\breve{D}\setminus Z_5\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{\left|\breve{D}\setminus Z_6\right|}-2^{\left|\breve{D}\setminus Z_6\right|}\right)\cdot 3^{\left|X\setminus\breve{D}\right|}+\\
& +\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_4|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_4|}+\\
& +\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_3|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_2|}+\\
& +\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot 3^{|X\setminus Z_1|}+\left(2^{|Z_3|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot 3^{|X\setminus Z_1|}
\end{aligned}
$$

Lemma 4.4. Let $\in \Sigma\ (\quad)$ and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_4\right)\right|$ my be calculated by the formula

$$
\begin{aligned}
\left|I^*\left(Q_4\right)\right| = {} & \left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\\
& \cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_5|}-1\right)\cdot\\
& \cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_4\right|}-3^{\left|\breve{D}\setminus Z_4\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_3\right|}-3^{\left|\breve{D}\setminus Z_3\right|}\right)\cdot\\
& \cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\\
& \cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_4|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_2\right|}-3^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_4|}-1\right)\cdot\\
& \cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\left(2^{|Z_3|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{\left|\breve{D}\setminus Z_1\right|}-3^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+\\
& +\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_2|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\\
& \cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\\
& \cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_2|}+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot 4^{|X\setminus Z_1|}
\end{aligned}
$$

Lemma 4.5. Let $\in \Sigma\ (\quad)$ and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_5\right)\right|$ my be calculated by the formula

$$\left|I^*\left(Q_5\right)\right| = \left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}$$

Lemma 4.6. Let $\in\Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_6\right)\right|$ my be calculated by the formula

$$\left|I^*\left(Q_6\right)\right| = \left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|X\setminus Z_4|}+\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+$$
$$+\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}+\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}$$

Lemma 4.7. Let $\in\Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_7\right)\right|$ my be calculated by the formula

$$\left|I^*\left(Q_7\right)\right| = \left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_6|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus Z_1|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+\left(2^{|Z_4|}-1\right)\cdot 2^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}$$

Lemma 4.8. Let $\in\Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_8\right)\right|$ my be calculated by the formula

$$\left|I^*\left(Q_8\right)\right| = \left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot$$
$$\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot 3^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot$$
$$\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

Lemma 4.9. Let $\in\Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I^*\left(Q_9\right)\right|$ my be calculated by the formula

$$\left|I^*\left(Q_9\right)\right| = \left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot$$
$$\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|};$$

Lemma 4.10. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{10})\right|$ my be calculated by the formula

$$\begin{aligned}\left|I^*(Q_{10})\right| = &\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D} \setminus Z_4|}-4^{|\breve{D} \setminus Z_4|}\right)\cdot 5^{|X \setminus \breve{D}|}+\\ &+\left(2^{|Z_6 \setminus Z_3|}-1\right)\cdot\left(2^{|Z_3 \setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D} \setminus Z_1|}-4^{|\breve{D} \setminus Z_1|}\right)\cdot 5^{|X \setminus \breve{D}|}+\\ &+\left(2^{|Z_4 \setminus Z_3|}-1\right)\cdot\left(2^{|Z_3 \setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D} \setminus Z_1|}-4^{|\breve{D} \setminus Z_1|}\right)\cdot 5^{|X \setminus \breve{D}|}+\\ &+\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(5^{|Z_2 \setminus Z_4|}-4^{|Z_2 \setminus Z_4|}\right)\cdot 5^{|X \setminus Z_2|}+\\ &+\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(5^{|Z_1 \setminus Z_4|}-4^{|Z_1 \setminus Z_4|}\right)\cdot 5^{|X \setminus Z_1|}+\\ &+\left(2^{|Z_4 \setminus Z_3|}-1\right)\cdot\left(2^{|Z_3 \setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D} \setminus Z_1|}-4^{|\breve{D} \setminus Z_1|}\right)\cdot 5^{|X \setminus \breve{D}|}\end{aligned}$$

Lemma 4.11. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{11})\right|$ my be calculated by the formula

$$\begin{aligned}\left|I^*(Q_{11})\right| = &\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(5^{|Z_2 \setminus Z_4|}-4^{|Z_2 \setminus Z_4|}\right)\cdot\left(6^{|\breve{D} \setminus Z_2|}-5^{|\breve{D} \setminus Z_2|}\right)\cdot 6^{|X \setminus \breve{D}|}\\ &+\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(5^{|Z_1 \setminus Z_4|}-4^{|Z_1 \setminus Z_4|}\right)\cdot\left(6^{|\breve{D} \setminus Z_1|}-5^{|\breve{D} \setminus Z_1|}\right)\cdot 6^{|X \setminus \breve{D}|}\end{aligned}$$

Lemma 4.12. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{12})\right|$ my be calculated by the formula

$$\begin{aligned}\left|I^*(Q_{12})\right| = &\left(2^{|Z_6 \setminus Z_3|}-1\right)\cdot\left(2^{|Z_5 \setminus Z_6|}-1\right)\cdot\left(3^{|Z_3 \setminus Z_4|}-2^{|Z_3 \setminus Z_4|}\right)\cdot 6^{|X \setminus Z_1|}+\\ &+\left(2^{|Z_3 \setminus Z_2|}-1\right)\cdot\left(2^{|Z_6 \setminus Z_3|}-1\right)\cdot\left(3^{|Z_2 \setminus Z_1|}-2^{|Z_2 \setminus Z_1|}\right)\cdot 6^{|X \setminus \breve{D}|}+\\ &+\left(2^{|Z_3 \setminus Z_2|}-1\right)\cdot\left(2^{|Z_4 \setminus Z_3|}-1\right)\cdot\left(3^{|Z_2 \setminus Z_1|}-2^{|Z_2 \setminus Z_1|}\right)\cdot 6^{|X \setminus \breve{D}|}\end{aligned}$$

Lemma 4.13. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{13})\right|$ my be calculated by the formula

$$\begin{aligned}\left|I^*(Q_{13})\right| = &\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3 \cap Z_2) \setminus Z_5|}\cdot\left(3^{|Z_3 \setminus Z_4|}-2^{|Z_3 \setminus Z_4|}\right)\cdot\\ &\cdot\left(3^{|Z_4 \setminus Z_2|}-2^{|Z_4 \setminus Z_2|}\right)\cdot\left(4^{|Z_2 \setminus Z_1|}-3^{|Z_2 \setminus Z_1|}\right)\cdot 7^{|X \setminus \breve{D}|};\end{aligned}$$

Lemma 4.14. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{14})\right|$ my be calculated by the formula

$$\left|I^*(Q_{14})\right| = \left(2^{|Z_5 \setminus Z_4|}-1\right)\cdot\left(2^{|Z_6 \setminus Z_5|}-1\right)\cdot\left(3^{|Z_4 \setminus Z_3|}-2^{|Z_4 \setminus Z_3|}\right)\cdot\left(7^{|\breve{D} \setminus Z_1|}-6^{|\breve{D} \setminus Z_1|}\right)\cdot 7^{|X \setminus \breve{D}|};$$

Lemma 4.15. Let $\in \Sigma$ ( ) and $Z_7 = \varnothing$. If X is a finite set, then the number $\left|I^*(Q_{15})\right|$ my be calculated by the formula

$$\left|I^*(Q_{15})\right| = \left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot$$
$$\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 4.16. Let $\in \Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I^*(Q_{16})\right|$ my be calculated by the formula

$$\left|I^*(Q_{16})\right| = \left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot$$
$$\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}$$

Theorem 4.2. Let $\in \Sigma$ ( ) and $Z_7=\varnothing$. If X is a finite set, then the number $\left|I(D)\right|$ my be calculated by the formula $\left|I(D)\right| = \sum_{i=1}^{16}\left|I^*(Q_i)\right|$.

5. Maximal Subgroups of the Semigroup $B_X(D)$ Defined by Semilatticesof The Class $\sum_3(X,8)$ In the given paper we give a full description maximal subgroups of the Complete semigroups of binary elations defined by semilattices of the class Σ ( ).

Lemma 5.1. The number of automorphisms of those semilattices, which are defined by the diagrams 1), 2), 3), 4), 5), 12), 13), 14) and 16) in fig. 3.4 is equal to 1, those semilattices which are defined diagrams 6), 7), 8), 9), 10) and 11) in fig. 3.4 is equal to 2 and that semilattice which is defined by the diagram 15) in fig. 3.4 is equal to 4.

Theorem 5.1. For any idempotent binary relation ε of the semigroup $B_X(D)$ the order a subgroup $G_X(D,\varepsilon)$ of the semigroup $B_X(D)$ is one, or two, or four.

6. Regular Elements of the Semigroup $B_X(D)$ defined by Semilattices of the Class $\Sigma_3(X,8)$. In this section we investigate the regular elements of the complete semigroups of binary relations defined by semilattices of the class $\Sigma_3(X,8)$.

Theorem 6.1. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. Then a binary relation α of the semigroup $B_X(D)$ that has a quasinormal representation of the form to be given below is a regular element of this semigroup iff there exist a complete $\alpha-$isomorphism φ of the semilattice $V(D,\alpha)$ on some subsemilattice D' of the semilattice D that satisfies at least one of the following conditions:

1) $\alpha = X \times T$, where $T \in D$;

2) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right)$, where $T, T' \in D$, $T \subset T'$, $Y_T^\alpha, Y_{T'}^\alpha \notin \{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha \supseteq \varphi(T)$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$;

3) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right)$, where $T, T', T'' \in D$, $T \subset T' \subset T''$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha \notin \{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap (\varphi) \neq \varnothing$;

4) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T'''}^\alpha \times T'''\right)$, where $T, T', T'', T''' \in D$, $T \subset T' \subset T'' \subset T'''$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$, and satisfies the conditions: $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_{T'''}^\alpha \cap \varphi(T''') \neq \varnothing$;

5) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where $Z_7 \subset T \subset T' \subset T'' \subset \breve{D}$, $Y_7^\alpha, Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_0^\alpha \notin \{\varnothing\}$, and satisfies the conditions: $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_T^\alpha \supseteq \varphi(T)$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $\boldsymbol{Y_T^\alpha \cap \varphi(T) \neq \varnothing}$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_0^\alpha \cap \breve{D} \neq \varnothing$;

6) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T' \cup T''}^\alpha \times (T' \cup T'')\right)$, where $T, T', T'' \in D$, $T \subset T'$, $T \subset T''$, $T' \setminus T'' \neq \varnothing$, $T'' \setminus T' \neq \varnothing$, $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$;

7) $\alpha = \left(Y_T^\alpha \times T\right) \cup \left(Y_{T'}^\alpha \times T'\right) \cup \left(Y_{T''}^\alpha \times T''\right) \cup \left(Y_{T'''}^\alpha \times T'''\right) \cup \left(Y_{T'' \cup T'''}^\alpha \times (T'' \cup T''')\right)$, where, $T \subset T' \subset T''$, $T \subset T' \subset T'''$, $T'' \setminus T''' \neq \varnothing$, $T''' \setminus T'' \neq \varnothing$ $Y_T^\alpha, Y_{T'}^\alpha, Y_{T''}^\alpha, Y_{T'''}^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha \supseteq \varphi(T)$, $Y_T^\alpha \cup Y_{T'}^\alpha \supseteq \varphi(T')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T''}^\alpha \supseteq \varphi(T'')$, $Y_T^\alpha \cup Y_{T'}^\alpha \cup Y_{T'''}^\alpha \supseteq \varphi(T''')$ $Y_{T'}^\alpha \cap \varphi(T') \neq \varnothing$, $Y_{T''}^\alpha \cap \varphi(T'') \neq \varnothing$, $Y_{T'''}^\alpha \cap \varphi(T''') \neq \varnothing$.

8) $\alpha = \left(Y_7^\alpha \times Z_7\right) \cup \left(Y_T^\alpha \times T\right) \cup \left(Y_4^\alpha \times Z_4\right) \cup \left(Y_2^\alpha \times Z_2\right) \cup \left(Y_1^\alpha \times Z_1\right) \cup \left(Y_0^\alpha \times \breve{D}\right)$, where $T \in \{Z_6, Z_5\}$, $Y_7^\alpha, Y_T^\alpha, Y_4^\alpha, Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \supseteq Z_7$, $Y_7^\alpha \cup Y_T^\alpha \supseteq \varphi(T)$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \supseteq Z_4$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_7^\alpha \cup Y_T^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$, $Y_T^\alpha \cap \varphi(T) \neq \varnothing$, $Y_4^\alpha \cap Z_4 \neq \varnothing$ $Y_2^\alpha \cap Z_2 \neq \varnothing$, $Y_1^\alpha \cap Z_1 \neq \varnothing$;

9) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times \breve{D}\right)$,
where $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

10) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)$,
where $T\subset T'$ $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T'\cup T''\subset T'''$ $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi\left(T'\right)$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi\left(T''\right)$, $Y_{T'}^\alpha\cap\varphi\left(T'\right)\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi\left(T''\right)\neq\varnothing$, $Y_{T'''}^\alpha\cap\varphi\left(T'''\right)\neq\varnothing$;

11) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_T^\alpha\times T\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$,
where $T\in\{Z_2,Z_1\}$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_T^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions:, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$ $Y_7^\alpha\cup Y_6^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_T^\alpha\supseteq\varphi\left(T\right)$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_5^\alpha\cap Z_5\neq\varnothing$ $Y_T^\alpha\cap\varphi\left(T\right)\neq\varnothing$, $Y_0^\alpha\cap\breve{D}\neq\varnothing$;

12) $\alpha=\left(Y_T^\alpha\times T\right)\cup\left(Y_{T'}^\alpha\times T'\right)\cup\left(Y_{T''}^\alpha\times T''\right)\cup\left(Y_{T'\cup T''}^\alpha\times\left(T'\cup T''\right)\right)\cup\left(Y_{T'''}^\alpha\times T'''\right)\cup\left(Y_{T'\cup T''\cup T'''}^\alpha\times\left(T'\cup T''\cup T'''\right)\right)$,
where $T\subset T'$, $T\subset T''$, $T'\setminus T''\neq\varnothing$, $T''\setminus T'\neq\varnothing$, $T''\subset T'''$, $\left(T'\cup T''\right)\setminus T'''\neq\varnothing$, $T'''\setminus\left(T'\cup T''\right)\neq\varnothing$, , $Y_T^\alpha,Y_{T'}^\alpha,Y_{T''}^\alpha,Y_{T'''}^\alpha,Y_{T'\cup T''\cup T'''}^\alpha\notin\{\varnothing\}$ and satisfies the conditions:, $Y_T^\alpha\cup Y_{T'}^\alpha\supseteq\varphi\left(T'\right)$, $Y_T^\alpha\cup Y_{T''}^\alpha\supseteq\varphi\left(T''\right)$ $Y_T^\alpha\cup Y_{T''}^\alpha\cup Y_{T'''}^\alpha\supseteq\varphi\left(T'''\right)$, $Y_{T'}^\alpha\cap\varphi\left(T'\right)\neq\varnothing$, $Y_{T''}^\alpha\cap\varphi\left(T''\right)\neq\varnothing$ $Y_{T'''}^\alpha\cap\varphi\left(T'''\right)\neq\varnothing$;

13) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$,
where $Z_5\subset Z_3$, $Z_5\subset Z_4$, $Z_3\setminus Z_4\neq\varnothing$, $Z_4\setminus Z_3\neq\varnothing$, $Z_4\subset Z_2$, $Z_1\setminus Z_2\neq\varnothing$, $Z_2\setminus Z_1\neq\varnothing$, , $Y_7^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_2^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\supseteq Z_7$, $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$ $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\supseteq Z_4$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_4^\alpha\cup Y_1^\alpha\supseteq Z_1$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$, $Y_4^\alpha\cap Z_4\neq\varnothing$ $Y_1^\alpha\cap Z_1\neq\varnothing$;

14) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_3^\alpha\times Z_3\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$,
where, $Z_6\subset Z_4$, $Y_7^\alpha,Y_6^\alpha,Y_5^\alpha,Y_4^\alpha,Y_3^\alpha,Y_1^\alpha,Y_0^\alpha\notin\{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha\cup Y_5^\alpha\supseteq Z_5$, $Y_7^\alpha\cup Y_6^\alpha\supseteq Z_6$, $Y_7^\alpha\cup Y_5^\alpha\cup Y_3^\alpha\supseteq Z_3$, $Y_5^\alpha\cap Z_5\neq\varnothing$, $Y_6^\alpha\cap Z_6\neq\varnothing$, $Y_3^\alpha\cap Z_3\neq\varnothing$ $Y_0^\alpha\cap\breve{D}\neq\varnothing$

15) $\alpha=\left(Y_7^\alpha\times Z_7\right)\cup\left(Y_6^\alpha\times Z_6\right)\cup\left(Y_5^\alpha\times Z_5\right)\cup\left(Y_4^\alpha\times Z_4\right)\cup\left(Y_2^\alpha\times Z_2\right)\cup\left(Y_1^\alpha\times Z_1\right)\cup\left(Y_0^\alpha\times\breve{D}\right)$

where $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_2^\alpha, Y_1^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_7^\alpha \cup Y_6^\alpha \cup Y_5^\alpha \cup Y_4^\alpha \cup Y_1^\alpha \supseteq Z_1$ $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$ $Y_1^\alpha \cap Z_1 \neq \varnothing$;

16) $\alpha = (Y_7^\alpha \times Z_7) \cup (Y_6^\alpha \times Z_6) \cup (Y_5^\alpha \times Z\ ) \cup (Y_4^\alpha \times Z_4) \cup (Y_3^\alpha \times Z_3) \cup (Y_2^\alpha \times Z_2) \cup (Y_1^\alpha \times Z_1) \cup (Y_0^\alpha \times \breve{D})$,

where, $Y_7^\alpha, Y_6^\alpha, Y_5^\alpha, Y_4^\alpha, Y_3^\alpha, Y_2^\alpha, Y_1^\alpha, Y_0^\alpha \notin \{\varnothing\}$ and satisfies the conditions: $Y_7^\alpha \supseteq Z_7$ $Y_7^\alpha \cup Y_5^\alpha \supseteq Z_5$, $Y_7^\alpha \cup Y_6^\alpha \supseteq Z_6$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_3^\alpha \supseteq Z_3$, $Y_7^\alpha \cup Y_5^\alpha \cup Y_6^\alpha \cup Y_4^\alpha \cup Y_2^\alpha \supseteq Z_2$, $Y_5^\alpha \cap Z_5 \neq \varnothing$, $Y_6^\alpha \cap Z_6 \neq \varnothing$, $Y_3^\alpha \cap Z_3 \neq \varnothing$, $Y_2^\alpha \cap Z_2 \neq \varnothing$;

Lemma 6.1. Let $D = \{Z_7, Z_6, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\} \in \Sigma_3(X,8)$ and $Z_8 \neq \varnothing$. If X is a finite set, then $\left|R^*(Q_1)\right| = 8$.

Lemma 6.2. Let $D = \{Z_7, Z_6, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\} \in \Sigma_3(X,8)$ and $Z_7 \neq \varnothing$. If X is a finite set, then $\left|R^*(Q_2)\right| = 23 \cdot \left(2^{|\breve{D} \setminus Z_7|} - 1\right) \cdot 2^{|X \setminus \breve{D}|}$.

Lemma 6.3. Let $D = \{Z_7, Z_6, Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\} \in \Sigma_3(X,8)$ and $Z_7 \neq \varnothing$. If X is a finite set, then

$$
\begin{aligned}
\left|R^*(Q_3)\right| = {} & 31 \cdot \left(2^{|Z_1 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_1|} - 2^{|\breve{D} \setminus Z_1|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot \left(2^{|Z_2 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_2|} - 2^{|\breve{D} \setminus Z_2|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot \left(2^{|Z_3 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_3|} - 2^{|\breve{D} \setminus Z_3|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot \left(2^{|Z_4 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_4|} - 2^{|\breve{D} \setminus Z_4|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_5|} - 2^{|\breve{D} \setminus Z_5|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot \left(2^{|Z_6 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_6|} - 2^{|\breve{D} \setminus Z_6|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& + 31 \cdot 2^{|Z_1 \setminus Z_5|} \cdot \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_1|} - 2^{|\breve{D} \setminus Z_1|}\right) \cdot 3^{|X \setminus \breve{D}|} + \\
& - 31 \cdot 2^{|Z_1 \setminus Z_3|} \cdot \left(2^{|Z_3 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_1|} - 2^{|\breve{D} \setminus Z_1|}\right) \cdot 3^{|X \setminus \breve{D}|} - \\
& - 31 \cdot 2^{|Z_1 \setminus Z_4|} \cdot \left(2^{|Z_4 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_1|} - 2^{|\breve{D} \setminus Z_1|}\right) \cdot 3^{|X \setminus \breve{D}|} - \\
& - 31 \cdot 2^{|Z_2 \setminus Z_4|} \cdot \left(2^{|Z_4 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_2|} - 2^{|\breve{D} \setminus Z_2|}\right) \cdot 3^{|X \setminus \breve{D}|} - \\
& - 31 \cdot 2^{|Z_3 \setminus Z_5|} \cdot \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_3|} - 2^{|\breve{D} \setminus Z_3|}\right) \cdot 3^{|X \setminus \breve{D}|} - \\
& - 31 \cdot 2^{|Z_4 \setminus Z_5|} \cdot \left(2^{|Z_5 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_4|} - 2^{|\breve{D} \setminus Z_4|}\right) \cdot 3^{|X \setminus \breve{D}|} - \\
& - 31 \cdot 2^{|Z_4 \setminus Z_6|} \cdot \left(2^{|Z_6 \setminus Z_7|} - 1\right) \cdot \left(3^{|\breve{D} \setminus Z_4|} - 2^{|\breve{D} \setminus Z_4|}\right) \cdot 3^{|X \setminus \breve{D}|} -
\end{aligned}
$$

Lemma 6.4. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$\begin{aligned}
\left|R^*\left(Q_4\right)\right| = {} & 20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+20\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}\\
&-20\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 3^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_3|}\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_4\setminus Z_5|}\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-\\
&-20\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}
\end{aligned}$$

Lemma 6.5. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7\neq\varnothing$. If X is a finite set, then:

$$\begin{aligned}
\left|R^*\left(Q_5\right)\right| = {} & 5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\\
&\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+5\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot
\end{aligned}$$

$$
\begin{aligned}
&\cdot\left(4^{\left|Z_1\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+5\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\\
&\cdot\left(4^{\left|Z_2\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_2\right|}-4^{\left|\breve{D}\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+5\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot\left(3^{\left|Z_4\setminus Z_5\right|}-2^{\left|Z_4\setminus Z_5\right|}\right)\cdot\\
&\cdot\left(4^{\left|Z_1\setminus Z_4\right|}-3^{\left|Z_1\setminus Z_4\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+5\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot\left(3^{\left|Z_3\setminus Z_5\right|}-2^{\left|Z_3\setminus Z_5\right|}\right)\cdot\\
&\cdot\left(4^{\left|Z_1\setminus Z_3\right|}-3^{\left|Z_1\setminus Z_3\right|}\right)\cdot\left(5^{\left|\breve{D}\setminus Z_1\right|}-4^{\left|\breve{D}\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

Lemma 6.6. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$
\begin{aligned}
\left|R^*\left(Q_6\right)\right|=\;&20\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot\left(2^{\left|Z_1\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}+20\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_6\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}+\\
&+20\cdot\left(2^{\left|Z_6\setminus Z_5\right|}-1\right)\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot 4^{\left|X\setminus Z_4\right|}+20\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}+\\
&+20\cdot\left(2^{\left|Z_2\setminus Z_3\right|}-1\right)\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-10\cdot 2^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot 2^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot\\
&\cdot 4^{\left|X\setminus Z_1\right|}-10\cdot 2^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 2^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_2\setminus Z_1\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-10\cdot 2^{\left|Z_6\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot\\
&\cdot 2^{\left|Z_3\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}-10\cdot 2^{\left|Z_3\setminus Z_4\right|}\cdot\left(2^{\left|Z_5\setminus Z_6\right|}-1\right)\cdot 2^{\left|Z_6\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}-\\
&-10\cdot 2^{\left|Z_4\setminus Z_1\right|}\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 2^{\left|Z_3\setminus Z_1\right|}\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}-10\cdot 2^{\left|Z_3\setminus Z_1\right|}\cdot\left(2^{\left|Z_3\setminus Z_4\right|}-1\right)\cdot 2^{\left|Z_4\setminus Z_1\right|}\cdot\\
&\cdot\left(2^{\left|Z_6\setminus Z_3\right|}-1\right)\cdot 4^{\left|X\setminus Z_1\right|}-10\cdot 2^{\left|Z_2\setminus Z_1\right|}\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot 2^{\left|Z_3\setminus\breve{D}\right|}\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}-10\cdot 2^{\left|Z_3\setminus\breve{D}\right|}\cdot\\
&\cdot\left(2^{\left|Z_3\setminus Z_2\right|}-1\right)\cdot 2^{\left|Z_2\setminus Z_1\right|}\cdot\left(2^{\left|Z_4\setminus Z_3\right|}-1\right)\cdot 4^{\left|X\setminus\breve{D}\right|}
\end{aligned}
$$

Lemma 6.7. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$
\begin{aligned}
\left|R^*\left(Q_7\right)\right|=\;&14\cdot\left(2^{\left|Z_4\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+14\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_6\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+14\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+14\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_3\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_2\setminus Z_3\right|}-2^{\left|Z_2\setminus Z_3\right|}\right)\cdot\left(3^{\left|Z_3\setminus Z_2\right|}-2^{\left|Z_3\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}+\\
&+14\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_4\cap Z_3\right)\setminus Z_5\right|}\cdot\left(3^{\left|Z_4\setminus Z_3\right|}-2^{\left|Z_4\setminus Z_3\right|}\right)\cdot\left(3^{\left|Z_3\setminus Z_4\right|}-2^{\left|Z_3\setminus Z_4\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-
\end{aligned}
$$

$$
\begin{aligned}
&-7\cdot 2^{\left|Z_4\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot 3^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 3^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\
&-7\cdot 2^{\left|Z_4\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot 3^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 3^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\
&-7\cdot 2^{\left|Z_4\setminus Z_6\right|}\cdot\left(2^{\left|Z_6\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot 3^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(3^{\left\|Z_1\setminus Z_2\right\|}-2^{\left\|Z_1\setminus Z_2\right\|}\right)\cdot 3^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-\\
&-7\cdot 2^{\left|Z_4\setminus Z_5\right|}\cdot\left(2^{\left|Z_5\setminus Z_7\right|}-1\right)\cdot 2^{\left|\left(Z_2\cap Z_1\right)\setminus Z_4\right|}\cdot 3^{\left|Z_2\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_2\setminus Z_1\right|}-2^{\left|Z_2\setminus Z_1\right|}\right)\cdot 3^{\left|Z_1\setminus\breve{D}\right|}\cdot\left(3^{\left|Z_1\setminus Z_2\right|}-2^{\left|Z_1\setminus Z_2\right|}\right)\cdot 5^{\left|X\setminus\breve{D}\right|}-
\end{aligned}
$$

$$
\begin{aligned}
&-7\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-7\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}
\end{aligned}
$$

Lemma 6.8. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$
\begin{aligned}
\left|R^*(Q_8)\right|=4\cdot&\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\\
&\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+4\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\\
&\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.
\end{aligned}
$$

Lemma 6.9. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$
\left|R^*(Q_9)\right|=\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 6^{|X\setminus\breve{D}|}
$$

Lemma 6.10. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$
\begin{aligned}
R^*(Q_{10})=12\cdot&\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+12\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\
&+12\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-6\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-6\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-
\end{aligned}
$$

$$
\begin{aligned}
&-6\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-\\
&-6\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}.
\end{aligned}
$$

Lemma 6.11. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{11})\right| = 4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.$$

Lemma 6.12. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$R^*(Q_{12}) = 4\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 6^{|X\setminus Z_1|}+$$
$$+4\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$-4\cdot 2^{|Z_4\setminus(Z_6\cup Z_3)|}\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

Lemma 6.13. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{13})\right| = \left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 6.14. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{14})\right| = \left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 6.15. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{15})\right| = 2\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 6.16. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set, then $\left|R^*(Q_{16})\right| = 2\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot$ $\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}$

Let us assume that $r=\sum_{i=1}^{16}\left|R^*(Q_i)\right|$

Theorem 6.2. Let $D\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set and R_D is a set of all regular elements of the semigroup $B_X(D)$. Then $|R_D|=r$.

7. Regular Elements of the Semigroup $B_X(D)$ defined by Semilattices of the Class $\Sigma_3(X,8)$ when $Z_7=\varnothing$. In this section we investigate the regular elements of the complete semigroups of binary relations defined by semilattices of the class $\Sigma_3(X,8)$ when $Z_7=\varnothing$.

Lemma 7.1. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then $\left|R^*(Q_1)\right|=1$.

Lemma 7.2. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then $\left|R^*(Q_2)\right|=7\cdot\left(2^{|\breve{D}|}-1\right)\cdot 2^{|X\setminus\breve{D}|}$

Lemma 7.3. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\begin{aligned}
\left|R^*(Q_3)\right| &= 16\cdot\left(2^{|Z_1|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}+16\cdot\left(2^{|Z_2|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+16\cdot\left(2^{|Z_3|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}+16\cdot\left(2^{|Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+16\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_5|}-2^{|\breve{D}\setminus Z_5|}\right)\cdot 3^{|X\setminus\breve{D}|}+16\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_6|}-2^{|\breve{D}\setminus Z_6|}\right)\cdot 3^{|X\setminus\breve{D}|}+\\
&+16\cdot 2^{|Z_1\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_1\setminus Z_3|}\cdot\left(2^{|Z_3|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_1\setminus Z_4|}\cdot\left(2^{|Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_1|}-2^{|\breve{D}\setminus Z_1|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_2\setminus Z_4|}\cdot\left(2^{|Z_4|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_2|}-2^{|\breve{D}\setminus Z_2|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_3|}-2^{|\breve{D}\setminus Z_3|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}-\\
&-16\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|\breve{D}\setminus Z_4|}-2^{|\breve{D}\setminus Z_4|}\right)\cdot 3^{|X\setminus\breve{D}|}
\end{aligned}$$

Lemma 7.4. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\begin{aligned}
\left|R^*(Q_4)\right| &= 15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+\\
&+15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_6|}-2^{|Z_2\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+
\end{aligned}$$

$$+15\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_6|}-2^{|Z_1\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_4|}-3^{|\breve{D}\setminus Z_4|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_3|}-3^{|\breve{D}\setminus Z_3|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_5|}-2^{|Z_2\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_5|}-2^{|Z_1\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_4\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}+$$
$$+15\cdot\left(2^{|Z_3\setminus Z_7|}-1\right)\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 3^{|Z_6\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 3^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_4|}\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_3|}\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_4\setminus Z_5|}\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_2\setminus Z_2|}\cdot\left(3^{|Z_2\setminus Z_4|}-2^{|Z_2\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_2|}-3^{|\breve{D}\setminus Z_2|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_4\setminus Z_5|}\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_4|}-2^{|Z_1\setminus Z_4|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}-$$
$$-15\cdot 2^{|Z_3\setminus Z_5|}\cdot\left(2^{|Z_5\setminus Z_7|}-1\right)\cdot 3^{|Z_1\setminus Z_1|}\cdot\left(3^{|Z_1\setminus Z_3|}-2^{|Z_1\setminus Z_3|}\right)\cdot\left(4^{|\breve{D}\setminus Z_1|}-3^{|\breve{D}\setminus Z_1|}\right)\cdot 4^{|X\setminus\breve{D}|}$$

Lemma 7.5. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*\left(Q_5\right)\right|=5\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_2\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$

$$+5\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+5\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_2\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+5\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_4|}-3^{|Z_1\setminus Z_4|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+5\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_3\setminus Z_5|}-2^{|Z_3\setminus Z_5|}\right)\cdot\left(4^{|Z_1\setminus Z_3|}-3^{|Z_1\setminus Z_3|}\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}$$

Lemma 7.6. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set,then

$$\begin{aligned}\left|R^*(Q_6)\right|&=10\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot\left(2^{|Z_1\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}+20\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\&+10\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_4|}+20\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}+\\&+10\cdot\left(2^{|Z_2\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-\\&-5\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\&-5\cdot 2^{|Z_1\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus\breve{D}|}\cdot\left(2^{|Z_2\setminus Z_1|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-\\&-5\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\&-5\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\&-5\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\&-5\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 4^{|X\setminus Z_1|}-\\&-5\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 4^{|X\setminus\breve{D}|}-\\&-5\cdot 2^{|Z_3\setminus\breve{D}|}\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot 2^{|Z_2\setminus Z_1|}\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot 4^{|X\setminus\breve{D}|}.\end{aligned}$$

Lemma 7.7. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\begin{aligned}\left|R^*(Q_7)\right|&=10\cdot\left(2^{|Z_4|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\&+10\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_6|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}+\\&+10\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_4\cap Z_3)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+\end{aligned}$$

$$-5\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$

$$-5\cdot 2^{|Z_4\setminus Z_6|}\cdot\left(2^{|Z_6|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{\|Z_1\setminus Z_2\|}-2^{\|Z_1\setminus Z_2\|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_1\setminus Z_2|}-2^{|Z_1\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_4\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_3|}-2^{|Z_2\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_1)\setminus Z_5|}\cdot 3^{|Z_2\setminus\breve{D}|}\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 3^{|Z_1\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_5\setminus Z_5|}\cdot\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_2\cap Z_3)\setminus Z_5|}\cdot 3^{|Z_2\setminus Z_1|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 3^{|Z_3\setminus\breve{D}|}\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}$$

Lemma 7.8. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*\left(Q_8\right)\right|=4\cdot\left(2^{|Z_6|}-1\right)\cdot\left(3^{|Z_4\setminus Z_6|}-2^{|Z_4\setminus Z_6|}\right)\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot$$
$$\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+4\cdot\left(2^{|Z_5|}-1\right)\cdot\left(3^{|Z_4\setminus Z_5|}-2^{|Z_4\setminus Z_5|}\right)\cdot$$
$$\cdot 3^{|(Z_2\cap Z_1)\setminus Z_4|}\cdot\left(4^{|Z_1\setminus Z_2|}-3^{|Z_1\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.$$

Lemma 7.9. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*\left(Q_9\right)\right|=\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_4)\setminus Z_5|}\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

Lemma 7.10. Let $D=\left\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\right\}\in\Sigma_3\left(X,8\right)$ and $Z_7=\varnothing$. If X is a finite set, then

$$R^*\left(Q_{10}\right)=10\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_4|}-4^{|\breve{D}\setminus Z_4|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+10\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$+10\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}+$$
$$-5\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_3\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot 2^{|Z_6\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_1|}-4^{|\breve{D}\setminus Z_1|}\right)\cdot 5^{|X\setminus\breve{D}|}-$$
$$-5\cdot 2^{|Z_3\setminus Z_1|}\cdot\left(2^{|Z_3\setminus Z_4|}-1\right)\cdot 2^{|Z_4\setminus Z_1|}\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(5^{|\breve{D}\setminus Z_2|}-4^{|\breve{D}\setminus Z_2|}\right)\cdot 5^{|X\setminus\breve{D}|}.$$

Lemma 7.11. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{11})\right|=4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_2\setminus Z_4|}-4^{|Z_2\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_2|}-5^{|\breve{D}\setminus Z_2|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+4\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot\left(5^{|Z_1\setminus Z_4|}-4^{|Z_1\setminus Z_4|}\right)\cdot\left(6^{|\breve{D}\setminus Z_1|}-5^{|\breve{D}\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}.$$

Lemma 7.12. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$R^*(Q_{12})=3\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot 6^{|X\setminus Z_1|}+$$
$$+3\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$+3\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(2^{|Z_4\setminus Z_3|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}+$$
$$-3\cdot 2^{|Z_4\setminus(Z_6\cup Z_3)|}\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_3\setminus Z_2|}-1\right)\cdot\left(3^{|Z_2\setminus Z_1|}-2^{|Z_2\setminus Z_1|}\right)\cdot 6^{|X\setminus\breve{D}|}$$

Lemma 7.13. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{13})\right|=\left(2^{|Z_5|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_5|}\cdot\left(3^{|Z_4\setminus Z_3|}-2^{|Z_4\setminus Z_3|}\right)\cdot$$
$$\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(4^{|Z_2\setminus Z_1|}-3^{|Z_2\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 7.14. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{14})\right|=\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_4|}-2^{|Z_3\setminus Z_4|}\right)\cdot\left(7^{|\breve{D}\setminus Z_1|}-6^{|\breve{D}\setminus Z_1|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 7.15. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{15})\right|=2\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(2^{|Z_6\setminus Z_5|}-1\right)\cdot 4^{|(Z_1\cap Z_2)\setminus Z_4|}\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot\left(5^{|Z_1\setminus Z_2|}-4^{|Z_1\setminus Z_2|}\right)\cdot 7^{|X\setminus\breve{D}|};$$

Lemma 7.16. Let $D=\{Z_7,Z_6,Z_5,Z_4,Z_3,Z_2,Z_1,\breve{D}\}\in\Sigma_3(X,8)$ and $Z_7=\varnothing$. If X is a finite set, then

$$\left|R^*(Q_{16})\right|=2\cdot\left(2^{|Z_6\setminus Z_3|}-1\right)\cdot 2^{|(Z_3\cap Z_2)\setminus Z_4|}\cdot\left(2^{|Z_5\setminus Z_6|}-1\right)\cdot\left(3^{|Z_3\setminus Z_2|}-2^{|Z_3\setminus Z_2|}\right)\cdot\left(5^{|Z_2\setminus Z_1|}-4^{|Z_2\setminus Z_1|}\right)\cdot 8^{|X\setminus\breve{D}|}$$

Let us assume that $r=\sum_{i=1}^{16}\left|R^*(Q_i)\right|$

Theorem 7.2. Let $D\in\Sigma_3(X,8)$ and $Z_7\neq\varnothing$. If X is a finite set and R_D is a set of all regular elements of the semigroup $B_X(D)$. Then $|R_D|=r$.

APROVEMENT OF THE RESULTS OF THE THESIS

The outcomes of the research proposals have been published in 7 international journals, and have been reported at various international conferences and general seminars:

1. Of the international conference "MODERN ALGEBRA AND ITS APPLICATION"- ბათუმი, 2010 წ., $XI-$Subsemilattice of the Semilattice of class $\Sigma_3(X,8)$.
2. Of the international conference "MODERN ALGEBRA AND ITS APPLICATION"- ბათუმი, 2011 წ., Idempotent Elements of the Semigroups $B_X(D)$ defined by Semilattices of the Class $\Sigma_3(X,8)$.
3. General Seminars at Department of mathematics, HACETTEPE University, Ankara, TURKEY, 2016, ,,Complete Semigroups of Binary Relations Defined By The Semilattices of a Class $\Sigma_3(X,8)$".
4. General Seminars at Department of mathematics, CANAKKALE Onsekiz Mart University, Canakkale, TURKEY 2016. ,,Complete Semigroups of Binary Relations Defined By The Semilattices of a Class $\Sigma_3(X,8)$".

The results obtained were discussed at the scientific workshops of the mathematics department of Batumi State University.

APPENDIX

There are 3 appendix at the end of the dissertation work. First appendix contains the example, where the $\Sigma_3(X,8)$class semilettices are constructed, when$|X|=4$. In the second appendix, the examples in case of finite set X, the elements of idempotent and regular elements of $B_X(D)$ semigroup given for the class semilettices are presented and it is shown that theoretical and practical calculations coincide.

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